A fuzzy coherent rule mining algorithm

Chun-Hao Chen, Ai-Fang Li, Yeong-Chyi Lee

Abstract

In real-world applications, transactions usually consist of quantitative values. Many fuzzy data mining approaches have thus been proposed for finding fuzzy association rules with the predefined minimum support from the given quantitative transactions. However, the common problems of those approaches are that an appropriate minimum support is hard to set, and the derived rules usually expose common-sense knowledge which may not be interesting in business point of view. In this paper, an algorithm for mining fuzzy coherent rules is proposed for overcoming those problems with the properties of propositional logic. It first transforms quantitative transactions into fuzzy sets. Then, those generated fuzzy sets are collected to generate candidate fuzzy coherent rules. Finally, contingency tables are calculated and used for checking those candidate fuzzy coherent rules satisfy the four criteria or not. If yes, it is a fuzzy coherent rule. Experiments on the foodmart dataset are also made to show the effectiveness of the proposed algorithm.

1. Introduction

Data mining is most commonly used in attempts to derive useful information and extract useful patterns from large data sets or database for solving specific issue. One of the commonly used techniques is association rule mining which is an expression $X \rightarrow Y$, where $X$ and $Y$ are a set of items [2]. It means in the set of transactions, if all the items in $X$ exist in a transaction, then $Y$ is also in the transaction with a high probability. For example, assume whenever customers in a supermarket buy bread and butter, they will also buy milk. From the transactions kept in the supermarkets, an association rule such as “Bread and Butter $\rightarrow$ Milk” will be mined out.

Lots of mining approaches are thus proposed for association rule mining [1,2,4,11], and most of them focused on binary valued transaction data. However, transaction data in real-world applications usually consist of quantitative values. Thus, by combing fuzzy theory, many mining algorithms have been proposed for deriving fuzzy rules from quantitative transaction database [3,5,13,14,16–19,25,26,29–31,37,39]. Some mining algorithms have also been applied on control problem and classification [4,21]. In [4], Kianmehri et al. adopt a multi-objective genetic algorithm based clustering method for determining and optimizing the membership functions of the fuzzy sets. In [21], Alcalà-Fdez et al. present a new fuzzy association rules classification method for high-dimensional problems.

However, there are two common problems of those fuzzy association rule mining approaches. The first one is that how to set appropriate minimum support and minimum confidence in fuzzy data mining. Obviously, it is a difficult task. If a large minimum support value is set, lots of potential rules may be deleted. On the contrary, large amount of rules will be derived such that decision makers cannot use them easily to make right decisions. The second problem is that some of those derived rules only expose common-sense knowledge and may not be interesting in business point of view. For example, if a rule “If milk is bought, Then bread is bought,” is derived with high support and confidence, it is a reliable rule according to Apriori algorithm [1]. But, it may not be valuable for business since the derived rule is common-sense knowledge, and it may mislead users because the rule “If milk is bought, Then bread is not bought” may also exist simultaneously.

Recently, Longbing Cao suggested the domain-driven data mining concept (D²M) [7,9,10], and cooperated it with industry knowledge to mine actual and useful information. Under the D²M concept, for those association rule mining algorithms on binary transaction [33], Sim et al. proposed a logical-based approach for deriving coherent rules. In that approach, by using the properties of propositional logic, relationship between items (also namely coherent rules) can be directly derived without knowing the appropriate value of minimum support.

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Thus, the main goal of this paper is attempted to derive actionable fuzzy rules from given quantitative transactions, which can also handle two common problems in fuzzy data mining. By utilizing the properties of propositional logic, the algorithm for mining fuzzy coherent rules without minimum support is proposed in this paper. The proposed approach first transforms quantitative transactions into fuzzy sets by the predefined membership functions. Candidate fuzzy coherent rules are then formed by the transformed fuzzy regions. For each candidate fuzzy coherent rule, the contingency table is then calculated according to the antecedent and consequent parts of that rule. At last, four criteria are used to judge the candidate fuzzy coherent rules. If a candidate rule satisfies the conditions, it is then a fuzzy coherent rule. So, if a fuzzy coherent rule “X → Y” is derived, then it means that the supports of “XY” and “¬X-Y” are both larger than supports of “¬XY” and “¬X¬Y”, which also means that the rule will not mislead users, more reliable, and could be provided to users for making correct marketing strategy. Experimental results on the foodmart dataset are performed to show the effectiveness of the proposed approach.

The remaining parts of this paper are organized as follows. Related work is stated in Section 2. The proposed fuzzy coherent rule mining algorithm is described in Section 3. An example to illustrate the proposed algorithm is given in Section 4. Experiments to demonstrate the performance of the proposed algorithm are stated in Section 5. Discussions are stated in Section 6. Conclusions and future works are given in Section 6.

2. Related work

In this section, related work will be introduced. In Section 2.1, the rule mining approaches, including binary and fuzzy data mining algorithms are described. The main issue of the minimum support threshold is stated in Section 2.2.

2.1. Binary and fuzzy data mining approaches

Data mining aims to extract useful knowledge and patterns from existing data to solve a specific issue. To date, it has been used in many different fields, such as shopping cart analysis [2], stock market analysis [3], and network intrusions [37]. The association rule which is the commonly used in shopping cart analysis, is represented by A → B, where A and B are products, and the rule expresses that if product A is purchased, product B will be purchased together with it. Two indexes are used to measure the validity of an association rule: support and confidence. The earliest association rule mining was suggested by Agrawal et al. [1], and the main three steps can be divided into: (1) produce candidate itemsets; (2) produce frequent itemsets based on minimum support; and (3) produce frequent itemsets based on minimum confidence.

In real-world application, since transactions always have quantitative values, thus how to handle the quantitative values becomes an interesting issue. Thus, by utilizing fuzzy theory, lots of mining algorithms have been proposed for deriving fuzzy rules from quantitative transaction database. The fuzzy data mining approaches are divided into two kinds, namely single-minimum-support fuzzy-mining (SSFM) [3,5,12,14,16–19,29–31,37,39] and multiple-minimum-support fuzzy-mining (MSFM) [25,26] approaches.

For SSFM approaches, Chan et al. proposed the F-APACS algorithm to mine fuzzy association rules [5]. They first transformed quantitative attribute values into linguistic terms and then used adjusted difference analysis to find interesting associations among the attributes. Hong et al. proposed a fuzzy mining algorithm to mine fuzzy rules from quantitative transaction data [17]. At nearly the same time, Kuok et al. proposed a fuzzy mining approach to handle numerical data in databases and to derive fuzzy association rules [19]. Basically, these fuzzy mining algorithms first use membership functions to transform each quantitative value into fuzzy sets and then use a fuzzy mining process to find fuzzy association rules. Yue et al. then extended the above concept to find fuzzy association rules with weighted items from transaction data [39]. They adopted Kohonen self-organized mapping to derive fuzzy sets for numerical attributes.

Regarding mining approaches for MSFM problems, Lee et al. proposed a mining algorithm that use multiple minimum supports to mine fuzzy association rules [25]. They assumed that items had different minimum supports and the minimum support for an itemset was set was the maximum of the minimum supports of items contained in the itemset. Under this constraint, the characteristics of level-by-level processing were kept, such that the original Apriori algorithm could easily be extended to find large itemsets. In [26], Lee et al. further extended the existing approach [25] and proposed a new fuzzy association rule mining algorithm with taxonomy.

2.2. The main issue of the minimum support threshold

The main issue of association rule mining approaches is how to define appropriate minimum support and minimum confidence, and the existed work has also reported that although the appropriate minimum support maybe exist, it is hard to find it [38]. As to how to find the appropriate minimum support, there are also lots of literatures have been published for this issue [33]. In general, using different minimum supports to derive association rules maybe result in different mining results. A small minimum support will generate too much frequent itemsets and association rules, which are not easily for user to make decisions. On the contrary, a larger minimum support will delete possible useful itemsets even they are infrequent and association rules [33]. In [22], Liu et al. thus proposed an algorithm that using multiple minimum supports, called Minimum Item Supports (MISs), for mining association rules. Lots of approaches have been then proposed approach for setting appropriate multiple minimum supports by using heuristics methods [20,23,40].

In order to solve those problems, Sim et al. thus proposed an association rule mining framework without minimum support threshold. In that approach, by using the properties of propositional logic, relationship between items can be derived directly without knowing the appropriate value of minimum support [33]. The main concept of that approach is that it maps the association rules to equivalences. And, each mapping from an association rule to an equivalence should satisfy conditions which are shown in Table 1.

From Table 1. X and Y are two itemsets. It show that an association rule X → Y is mapped to p←q, if and only if (1) X → Y is true; (2) ¬X → Y is false; (3) X → ¬Y is false; and (4) ¬X → ¬Y is true. When used in multiple transactions, it can map association rules to implications as follows: X → Y is mapped to an implication p ← q, if and only if (1) Sup(X, Y) > Sup(X, Y); (2) Sup(X, Y) > Sup¬X, Y; (3) Sup(X, Y) > Sup(X, ¬Y); and (4) Sup(X, Y) > Sup¬X, ¬Y. In the same way, others association rules that mapped to implications based on comparison between supports can be derived, and
Table 2

The contingency table of a rule.

<table>
<thead>
<tr>
<th>Antecedent X</th>
<th>Consequence Y</th>
<th>~Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>( Q_1 = \text{Sup}(X, Y) )</td>
<td>( Q_2 = \text{Sup}(X, ~Y) )</td>
</tr>
<tr>
<td>~X</td>
<td>( Q_3 = \text{Sup}(~X, Y) )</td>
<td>( Q_4 = \text{Sup}(~X, ~Y) )</td>
</tr>
</tbody>
</table>

3. The proposed fuzzy coherent rule mining algorithm

Based on the fuzzy data mining algorithm [17] and the coherent rule concept [33] which is described in above section, the proposed algorithm for mining coherent fuzzy rules is described below.

The proposed fuzzy coherent rule mining algorithm:

**INPUT:** A body of \( n \) quantitative transaction data, a given item \( y \), and a given set of membership functions.

**OUTPUT:** A set of fuzzy coherent rules (FCR).

**STEP 1:** Transform the quantitative value \( s_j^{(i)} \) of each transaction datum \( D_j^{(i)} \), \( i = 1 \) to \( n \), for each item \( i, j = 1 \) to \( m \), into fuzzy values represented as \( \left( f_h^{(i)}(R_j^h), f_{h+1}^{(i)}(R_j^{h+1}), \ldots, f_m^{(i)}(R_j^m) \right) \) using the given membership functions, where the fuzzy region \( R_j^k \) is defined as item.termm. It means the \( k \)th fuzzy region of item \( j \). The fuzzy value \( f_h^{(i)}(R_k^h) \) is \( s_j^{(i)} \)'s fuzzy membership value in region \( R_j^k \), \( k = 1 \) to \( l \), and \( l \) is the number of linguistic terms for \( j \). A fuzzy region is defined as item.termm.

**STEP 2:** For each fuzzy region \( R_k \), calculate its complement value according to the definition of fuzzy complement which is: If \( f_h^{(i)}(R_k^h) \) is the fuzzy value of \( k \)th fuzzy region for item \( j \), then its fuzzy complement is calculated as \( 1 - f_h^{(i)}(R_k^h) \). The result is represented as follows:

\[
1 - f_h^{(i)}(R_k^h) = \left( \frac{1}{R_{j_1}} + \frac{1}{R_{j_2}} + \cdots + \frac{1}{R_{j_l}} \right) \cdot f_h^{(i)}(R_k^h),
\]

where \( i \) is the transaction id number, the \( f_h^{(i)}(R_k^h) \) is \( s_j^{(i)} \)'s membership value in fuzzy region \( R_k \).

**STEP 3:** Collect all fuzzy regions into the set \( A \). In other words, the set \( A = \{ R_{j_1}, R_{j_2}, \ldots, R_{j_l}, R_{j_{l+1}}, \ldots, R_{j_m} \} \), where \( j = 1 \) to \( m \) and \( k = 1 \) to \( l \). \( m \) is the number of items and \( l \) is the number of linguistic terms in the given membership functions.

**STEP 4:** Collect all fuzzy regions of the given item \( y \) into the set \( L \). In other words, the set \( L = \{ R_{y_1}, R_{y_2}, \ldots, R_{y_k} \} \).

**STEP 5:** Remove the given item \( y \)'s fuzzy regions from the set \( A \) to from the set \( K \). In other words, the set \( K = A \setminus L \). The two sets \( L \) and \( K \) are then used to generate the candidate coherent fuzzy rules in the following steps.

STEP 6: Set \( h = 1 \), where \( h \) means the length of antecedent \( X \) and form candidate fuzzy coherent rule \( X \rightarrow Y \), where \( X \) is an element of \( K \), and \( Y \) is an element of \( L \).

**STEP 7:** Do the following substeps to generate fuzzy coherent rules:

1. **SUBSTEP 7.1:** Calculate the contingency table for antecedent \( X \) and consequent \( Y \). Here, four count values will be calculated, including \( Q_1: \text{count}_{XY}, Q_2: \text{count}_{X~Y}, Q_3: \text{count}_{~X~Y} \), and \( Q_4: \text{count}_{~X~Y} \). Each of them is calculated as follows:

\[
\text{count}_X = \sum_{i=1}^{m} f_s^{(i)},
\]

where the fuzzy value of an itemset \( S \) in each transaction is calculated as \( f_s^{(i)} = f_{h_1}^{(i)}(A_{h_1}) \ldots A_{h_l}^{(i)}(A_{h_l + 1}) \), and \( f_s^{(i)} \) is the membership value of fuzzy item \( S \) in ith transaction. If the minimum operator is used for the intersection, then:

\[
f_s^{(i)} = \min_{j=1}^{m} f_s^{(i)}.
\]

**SUBSTEP 7.2:** Check the candidate fuzzy coherent rule meets the four conditions, which are \( Q_1 > Q_2, Q_3 > Q_2, Q_4 > Q_2 \) or not. If yes, let \( FCR_{all} = FCR_J(X), Y) \), go to **SUBSTEP 7.1** to calculate the contingency table of next candidate fuzzy coherent rule. If all candidate rules are checked, go to **SUBSTEP 7.3**. Otherwise, go to **STEP 8**.

**SUBSTEP 7.3:** Check the \( FCR_h \) is empty or not. If yes, go to **STEP 11**. Otherwise, go to **STEP 8**.

**STEP 8:** Collect the fuzzy regions of antecedent and consequent parts of the derived fuzzy coherent rules in \( FCR_h \) to form new set \( K \) and \( L \), respectively.

**STEP 9:** Set \( h = h + 1 \).

**STEP 10:** Form candidate fuzzy coherent rule \( X \rightarrow Y \) according to \( L \) and \( K \), where the length of \( X \) is \( h \). Note that fuzzy regions with the same item cannot be used to form candidate fuzzy coherent rules.

**STEP 11:** If there is no candidate fuzzy coherent rule, go to **STEP 12**. Otherwise, go to **STEP 7.1**.

**STEP 12:** Output the derived fuzzy coherent rules \( FCR_{all} \).

Note that the proposed approach can easily extend to mine all fuzzy coherent rules through repeating **STEP 1** to **STEP 12** using different item \( Y \).

4. An example of the proposed algorithm

In this section, an example is given to illustrate the proposed mining algorithm. This is a simple example to show how the proposed algorithm can be used to mine fuzzy coherent rules from quantitative transaction data. Assume there are four items in a transaction database: milk, bread, cookies, and beverage. The data set includes the six transactions shown in **Table 3**.

Assume the fuzzy membership functions for the four items are as shown in **Fig. 1**.

In this example, each item has three fuzzy regions: Low, Medium, and High. Thus, three fuzzy membership values are produced for each item according to the predefined membership functions. For the transaction data in **Table 3**, the proposed mining algorithm proceeds as follows.

**Table 3**

Six transactions in this example.

<table>
<thead>
<tr>
<th>TID</th>
<th>Items</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1</td>
<td>(Milk, 10); (bread, 10); (cookies, 7); (beverage, 7)</td>
</tr>
<tr>
<td>T2</td>
<td>(Milk, 12); (bread, 14); (cookies, 12)</td>
</tr>
<tr>
<td>T3</td>
<td>(Bread, 2); (cookies, 12)</td>
</tr>
<tr>
<td>T4</td>
<td>(Milk, 2); (bread, 4); (cookies, 5)</td>
</tr>
<tr>
<td>T5</td>
<td>(Milk, 9); (bread, 9)</td>
</tr>
<tr>
<td>T6</td>
<td>(Milk, 5); (beverage, 12)</td>
</tr>
</tbody>
</table>

STEP 1: The quantitative value of each transaction datum is transformed into a fuzzy set according to the predefined membership functions. Take the second item in transaction 75 using the membership functions as an example. The amount “9” of item bread is then converted into the fuzzy set, \((0.75/\text{bread.Medium} + 0.25/\text{bread.High})\), using the given membership functions. The results for all items are shown in Table 4, where the notation item.term is called a fuzzy region.

STEP 2: The fuzzy value is transformed into a complement fuzzy set. Take the second item in transaction 75 using the membership functions as an example. The fuzzy region \((0.75/\text{bread.Medium} + 0.25/\text{bread.High})\) of item bread is then converted into the complement fuzzy set \((1/\text{bread.Low} + 0.25/\text{bread.Medium} + 0.75/\text{bread.High})\). The results for all items are shown in Table 5.

STEP 3: All fuzzy regions are then collected into a set \(A\), which is \(A=\{\text{milk.Low, milk.Medium, milk.High, bread.Low, bread.Medium, bread.High, cookies.Low, cookies.Medium, cookies.High, beverage.Low, beverage.Medium, beverage.High}\}\). 

STEP 4: The fuzzy regions of milk are collected to form the set \(L=\{\text{milk.low, milk.Medium, milk.High}\}\).

STEP 5: The set \(K\) is generated from the set \(A\) by removing fuzzy regions of milk, which is \(K=\{\text{bread.Low, bread.Medium, bread.High, cookies.Low, cookies.Medium, cookies.High, beverage.Low, beverage.Medium, beverage.High}\}\).
STEP 6: Let \( h = 1 \). The candidate fuzzy coherent rule set is generated as follows: \((\text{bread.Low} \rightarrow \text{milk.Low}), (\text{bread.Medium} \rightarrow \text{milk.Low}), (\text{bread.High} \rightarrow \text{milk.Low}), \ldots, (\text{beverage.High} \rightarrow \text{milk.High})\).

STEP 7: The fuzzy coherent rules are derived by the following substeps:

**SUBSTEP 7.1:** Take a candidate fuzzy coherent rule “(bread.Medium \( \rightarrow \) milk.Medium)” as an example. Since the consequence part is milk.Medium and the antecedent part is bread.Medium, the contingency table of (bread.Medium, milk.Medium) is then calculated. For instance, the count value of count(bread.Medium, milk.Medium) is 1.25 (=0.5 + 0 + 0.75 + 0.25) as shown in Table 6. By the same way, count(bread.Medium, ¬milk.Medium), count(¬bread.Medium, milk.Medium) and count(¬bread.Medium, ¬milk.Medium) of the contingency table for (bread.Medium, milk.Medium) are show in Table 7.

**SUBSTEP 7.2:** From Table 7, it can observe that (bread.Medium \( \rightarrow \) milk.Medium) meets the four conditions that are \( Q_1 > Q_2, Q_1 > Q_3, Q_2 > Q_3 \) and \( Q_4 > Q_3 \). Thus, (bread.Medium \( \rightarrow \) milk.Medium) is put into the FCR1 and FCR_all. Other candidate fuzzy coherent rules are processed in the same way.

**SUBSTEP 7.3:** After SUBSTEP 7.2, there are two fuzzy coherent rules are generated which are FCR1 = ((bread.Low \( \rightarrow \) milk.Low), (bread.High \( \rightarrow \) milk.High)). Since the FCR1 is not empty, it then goto STEP 8.

STEP 8: According to the derived FCR1, the \( L \) and \( K \) is renewed as \( L = \{\text{milk.Medium, milk.High}\} \) and \( K = \{\text{bread.Medium, bread.High}\} \).

STEP 9: The parameter \( h \) is set at 2.

STEP 10: Since there are only two fuzzy regions with the same item in the set \( K \), it cannot be used to form any candidate fuzzy coherent rule in this step.

STEP 11: Because there is no candidate rule be formed, it then go to STEP 12.

STEP 12: In this example, totally two fuzzy coherent rules are outputted. The derived FCR_all are shown in Table 8.

From Table 8, according to the properties of propositional logic, take the first fuzzy coherent rule as an example, it means that it is not only stronger than rules “bread.middle → ¬milk.middle” and “¬bread.middle → milk.middle” in terms of support, but also means that the middle amount of bread and the middle amount of milk are always bought together in the given transactions. So, such a rule could then provide more precise information to users for making marketing strategy.

5. Experiment results

In this section, experimental results of the proposed approach are described. They were implemented in Java on a personal computer with Intel Core i7, 2.93 GHz and 4 GB RAM. The dataset is described in Section 5.1. The performance evolutions are given in Section 5.2.

5.1. Dataset descriptions

A dataset, the foodmart database [28], was used to evaluate the performance of the algorithms under different comparisons. The foodmart database lay in the database product Microsoft SQL Server 2000. The related information of this dataset was listed as follows. There were 21,556 transactions; the total number of different items was 1600. In the following experiments, 1000 transactions of 21,556 transactions are used to evaluate the proposed approach. The membership functions are given in Fig. 2. There are three fuzzy set used in the experiments that are Low, Medium and High.

5.2. Experimental evaluations

Firstly, experiments were made to show the comparison results of the derived rules between the proposed approach (FCR) and the original fuzzy rule mining approach (FAR) [6]. The minimum support and minimum confidence of FAR is set at 0.000066 and 0.1, respectively. The results are shown in Table 9.

From Table 9, it can observe that the number of rules derived by FCR is 158 which is less than that by FAR when the length of a rule is 2. However, when the length of a rule is 4, the number of derived rules by FCR is larger than that by FAR. The proposed approach can derive extra eight rules than FAR. In addition, the average support and confidence of rules derived by the proposed approach is 0.0008 and 0.87, which are also larger than that by FAR. Those results show that the proposed approach is effective.
Table 9
Comparison results between FCR and FAR.

<table>
<thead>
<tr>
<th>The length of antecedent part in a rule</th>
<th>FCR</th>
<th>FAR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of rules (length = 2)</td>
<td>158</td>
<td>7240</td>
</tr>
<tr>
<td>Number of rules (length = 3)</td>
<td>42</td>
<td>435</td>
</tr>
<tr>
<td>Number of rules (length = 4)</td>
<td>8</td>
<td>0</td>
</tr>
<tr>
<td>Total number of rules</td>
<td>208</td>
<td>7675</td>
</tr>
<tr>
<td>The average confidence</td>
<td>0.870928</td>
<td>0.393698</td>
</tr>
<tr>
<td>The average support</td>
<td>0.00800481</td>
<td>0.000417557</td>
</tr>
</tbody>
</table>

Table 10
The statistics analyses between FAR and FCR.

<table>
<thead>
<tr>
<th></th>
<th>Appear in FAR and FCR</th>
<th>Appear only in FAR</th>
<th>Appear only in FCR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of support</td>
<td>32</td>
<td>7643</td>
<td>176</td>
</tr>
<tr>
<td>Average support</td>
<td>0.000875</td>
<td>0.000415641</td>
<td>0.000786932</td>
</tr>
<tr>
<td>Average confidence</td>
<td>0.836999</td>
<td>0.391842</td>
<td>0.877097</td>
</tr>
</tbody>
</table>

in finding more reliable rules. In order to explain the merits of the proposed approach more clearly, a derived fuzzy coherent rule with rule length equals 4 is given as follows.

“If 332L, 62M, and 671M, Then 349M, sup = 0.00075, conf = 1.0”

From the rule, although the support value of that rule is very small, its confidence value is 100% and may be useful in terms of business. However, the rule will be pruned by using FAR if the minimum support is larger than 0.00075. At last, the statistics analyses between rules derived by the proposed approach and FAR are given in Table 10.

From Table 10, those derived rules are divided into three groups. The first one is that rules that can derive by the proposed approach and the FAR. There are 32 rules, and it can be observed that although they have small average support value (0.000875), the average confidence value is very high (0.837). The second and third groups represent those rules only derived by using FAR and the proposed approach, respectively. The results show that the average support and confidence of rules derived by FAR which are 0.00041 and 0.39 are smaller than that by the proposed approach which are 0.00078 and 0.877.

In order to show more detail results, since the foodmart dataset has taxonomy, we choose items in the food category for the following experiments. Table 11 shows the comparison results of the derived rules between the FCR approach and FAR approach of food category.

From Table 11, we can observe that the number of derived rules by FCR is less than by FAR, which show the same phenomena with the previous experiments. The interesting part is that it can find a rule with length equals to four in food category by FCR when comparing with FAR as follows:

“If bought small amount of Better Vegetable Soup, medium amount of Gorilla Low Fat String Cheese, and medium amount of Club Blueberry Yogurt, Then bought medium amount of Swell Canned Mixed Fruit, sup = 0.00075, conf = 1.0”

Although the support of the derived fuzzy coherent rule is low, the confidence is 1.0 and it satisfies logical equivalence. Thus, such a rule can be recognized as a recipe for people who is on a diet. They usually make a fruit yogurt and a cup of cheese vegetable soup. Furthermore, we can thus get a pattern that those people who are on a diet like to buy these four items. We can also know that people who are on a diet always bought these items more than the others. In other words, the derived fuzzy coherent rule can not only emphasize certain customs or activities than fuzzy association rules, but also be used to forecast and focus on specific customer group on target-market decision. At last, the execution times of the proposed approach with different data size are shown in Table 12.

From Table 12, we can observe that the execution times of the mining process are increasing along with the increasing of the data size. From those experimental results, we can conclude that the proposed approach provides an interesting way to find association rules without minimum support threshold, and it is effective.

Table 11
The derived rules by FCR and FAR in food category.

<table>
<thead>
<tr>
<th></th>
<th>FCR</th>
<th>FAR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of rules (length = 2)</td>
<td>84</td>
<td>3232</td>
</tr>
<tr>
<td>Number of rules (length = 3)</td>
<td>18</td>
<td>183</td>
</tr>
<tr>
<td>Number of rules (length = 4)</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Total number of rules</td>
<td>103</td>
<td>3415</td>
</tr>
</tbody>
</table>

Table 12
The execution times of the proposed approach.

<table>
<thead>
<tr>
<th>Size of dataset</th>
<th>5k</th>
<th>10k</th>
<th>15k</th>
<th>21k</th>
</tr>
</thead>
<tbody>
<tr>
<td>Execution times (s)</td>
<td>49.89</td>
<td>102.58</td>
<td>157.86</td>
<td>237.47</td>
</tr>
</tbody>
</table>

6. Discussion

By using the properties of propositional logic, the proposed approach is utilized for mining the fuzzy coherent rules. The proposed approach solves two common problems in fuzzy data mining. The first one is how to set appropriate minimum support. The second one is how to derive more actionable rules in terms of business. Thus, if a fuzzy coherent rule “X → Y” is derived, it represents two things. The first one is that it can make sure the items in the fuzzy coherent rule are always purchased together in transactions. In other words, the supports of the both rules “X → Y” and “X → ¬Y” are larger than “¬X → Y” and “¬X → ¬Y”. And, it does not mislead users, which also means that the derived rule provides more precise information to decision maker for making more actionable marketing strategy. Currently, the proposed approach is a little time-consuming since it is based on Apriori-like algorithm (see Table 12). Assume the average transaction width is w, then the time complexity of finding fuzzy coherent rules with the given item Y is O \((\sum_{h=2}^{\infty} |K_{h-1}| \times |I| \times numberTran \times w)\), where \(|K_{h-1}|\) means that the number of fuzzy regions in the set K and the length of each element is \((h - 1)\), \(|I|\) is the number of fuzzy regions in the set \(|I|\), and \(numberTran\) is the transaction size. Thus, \((|K_{h-1}| \times |I|)\) is the time for generating candidate fuzzy coherent rules. For each rule, \(numberTran \times w\) is needed to check it is a fuzzy coherent rule or not. To speed up the mining process, some techniques can be used for achieving this purpose, e.g., parallel processing [15], projection technique [27], or sampling techniques [32], and we will put them in future work.

Besides, in the past decade, lots of recommendation algorithms have been proposed for e-commerce [24,32]. The main concept of them is attempted to find similar customers that have similar purchased behaviors. And, based on the purchased behaviors, appropriate items are recommended to new customers. Basically, they are divided into three types, including traditional collaborative filtering, cluster model and item-to-item collaborative filtering. Then, according to different types of recommendation algorithms, items that may interest to new customers are recommended. Though the proposed approach, the derived fuzzy coherent rules can be utilized as a part of the recommendation system. For instance, when we have the fuzzy coherent rule like follows:

“If bought small amount of Better Vegetable Soup, medium amount of Gorilla Low Fat String Cheese, and medium amount of Club Blueberry Yogurt, Then bought medium amount of Swell Canned Mixed Fruit, sup = 0.00075, conf = 1.0”

then if customers bought “Better Vegetable Soup”, “Gorilla Low Fat String Cheese” and “Club Blueberry Yogurt”, the item “Swell Canned...”
Mixed Fruit" could be recommended to him (her). In the same way, the proposed approach could also be applied to others fields, e.g., data stream, path planning, etc. [8,11,34–36].

7. Conclusion and future works

In this paper, we have proposed a fuzzy coherent rule mining algorithm from quantitative transactions without minimum support threshold. The proposed algorithm first transforms quantitative transactions into fuzzy sets by the predefined membership functions. Then, the fuzzy sets are collected to form candidate fuzzy coherent rules. The contingency table is then calculated for each candidate fuzzy coherent rule, and used to check whether it satisfies the four conditions or not. Experiments on the foodmart dataset have also been made to show the merits of the proposed approach. Firstly, they show that the proposed approach can derive more rules than the original fuzzy rule mining approach in terms of rule length. Secondly, they also show that the proposed approach can derive useful rules in terms of high confidence rules.

The main contribution of this work is that the proposed approach can derive interesting rules effectively without setting minimum support. In the future, we will continue to enhance the proposed approach to more complexity problems.

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References
