Yielding Pressure of Spread Footing above Multiple Voids

Makoto Kiyosumi1, Osamu Kusakabe2, Masatoshi Ohuchi3, and Fang Le Peng4

Abstract: The effect of multiple voids on the yielding pressure of strip footing was numerically investigated by a two-dimensional plane strain finite-element method analysis. The results indicated that the failure zone developed mainly towards the nearest void from the footing and did not generally extend to the other voids, and the failure zone was narrower and smaller than that of the no-void ground, resulting in smaller yielding pressure. A practical calculation formula was developed for estimating the yielding pressure of strip footing above multiple voids.

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CE Database subject headings: Voids; Finite element method; Foundation design; Spread foundations.

Introduction

Calcareous sediments are widely spread throughout tropical and subtropical regions. These sediments often pose difficult problems when constructing building foundations due to their unique geotechnical properties, such as high crushability and existence of voids of various sizes. Calcareous sediments are distributed over the Okinawa region in Japan, which are called Ryukyu calcareous sediments. The sediments composed of cemented and/or uncremented debris coral have a thickness of about 20–50 m overlying the base rock, called Shimajiri mudstone. The Ryukyu calcareous sediments have some unfavorable characteristics as the bearing stratum of shallow footing, which are: (1) highly irregular N-value distributions both in vertical and horizontal directions; (2) alternating layers of cemented and uncremented layers; and (3) numerous voids of various sizes (Uehara et al. 1988).

Recent construction projects in the Okinawa region, including bridges connecting islands and high rise buildings, necessitate appropriate foundation design strategies related to selection of the bearing stratum together with the type of foundation. Selection of the deeply situated Shimajiri mudstone for a bearing layer and possible pile foundations will face difficulties in excavating the hard and thick Ryukyu calcareous sediments and may not be economical, whereas there is no practical design method of shallow foundation founded on an alternating layer of calcareous sediment with multiple voids (Japan Road Association 2002). There has, therefore, been a need to establish a design strategy for the placement of shallow foundations that involve calcareous sediments with multiple voids. In this paper, the effect of multiple voids on the yielding pressure characteristics of the spread footing was examined by two-dimensional plane strain FEM analysis, and a practical calculation method was then deduced and proposed. One possible design and construction strategy was also demonstrated.

Review of Previous Studies

The effect of the presence of a single void on the bearing capacity of shallow footing has been studied both experimentally and numerically (Baus and Wang 1983; Badie and Wang 1984; Wang and Badie 1985; Al-Tabbaa et al. 1989; Azam et al. 1997). Baus and Wang (1983) performed 47 small scale model tests of shallow footing on a compacted silty clay in the plane strain condition, in which the footing was centered with continuous rectangular voids and was subjected to vertical central loading. Together with the model tests, they carried out two-dimensional FEM analyses with an elastic perfectly plastic material, demonstrating that FEM results agree well with the experimental load–settlement curves, and concluded that there exists a critical depth below which the presence of a void has a negligible influence on the bearing capacity. They summarized the results in a chart, which shows the effect of the void depth/footing width ratio and void width/footing width ratio on the bearing capacity. Their analysis also indicated that the void shape has a negligible effect on the bearing capacity by comparing the load–settlement curves of the footing for circular and square continuous voids, provided that the void depth/footing width ratio and the void width/footing width ratio are the same. Badie and Wang (1984) extended their FEM program to three dimensions and compared the FEM results with another series of small scale model tests of shallow footing on a compacted kaolinite clay. This particular paper focused on the square footing above a continuous circular void, wherein the footing was situated both on and off the center line of the void, with the results shown in a diagram of bearing capacity versus void depth/footing width ratio for various values of eccentricity of void from the footing, both for strip and square footings.
(1985) further extended their investigation to the situation in which a continuous void was used, which was either parallel to or perpendicular to the strip footing.

Al-Tabbaa et al. (1989) presented the results of small scale model tests of strip footing on a cement mixed sand with a continuous circular void subjected to vertical loading both for in-line and offset conditions. They observed that the deeper the void, the stiffer and stronger is the response, and that the influence of the shape of void is not very great, which are consistent with the earlier observations made by Baus and Wang (1983). The photographs provided show that two distinct vertical shear planes developed from the edges of the footing if the void is located on the centerline of the footing. Also, the two tilting shear planes extend toward the right shoulder of the void and the left-hand side of the void, when the location of the void is off to the right-hand side.

More recently, Azam et al. (1997) conducted two-dimensional plane-strain FEM analyses to study the effect of a void on the stability of strip footing in two layer soils where the bottom layer includes a continuous circular void. Based on analyses of 160 cases of different conditions of soils and void, they presented the diagrams of reduction factor $R$ (bearing capacity with void divided by that without void) versus void depth/void diameter ratio for various soil conditions, and deduced a practical equation for $R$ as a function of bearing capacity ratio of top and bottom layers, void depth/void diameter ratio and a ratio of top layer/footing width. As was reviewed above, to date no literature on the effect of multiple voids on the bearing capacity of shallow footing exists.

**Method of FEM Analysis**

Fig. 1 shows a typical finite-element mesh in the plane strain condition. The ground was assumed to be a uniform saturated layer with voids. The footing width ($B$) was taken to be 2 m and the area of analysis was 20 $B$ in width and 15 $B$ in depth, to minimize possible boundary effects. At the boundaries, the vertical and horizontal displacements at the bottom boundary were fixed, as were the horizontal displacements at the side boundaries. The footing base was assumed to be perfectly rough. The element stiffness matrix was evaluated by numerical integration using a total of 12 Gauss points. There are 511 elements, 4,233 nodes, and 6,132 Gauss points in the mesh shown in Fig. 1. In order to select the appropriate numbers of elements and nodes and Gauss points, these numbers were varied in the ranges of 259–1,210, 2,159–9,845, and 3,108–14,520, respectively, for the void case, and 291–1,216, 2,427–9,909, and 3,492–14,592 for the no-void case during the preliminary trial. The mesh arrangement in the vicinity of the footing and void was also varied. It was confirmed that the variation in the obtained results was negligible with respect to the yielding pressure and yielding settlement when the number of elements exceeds about 600. The program was used was a commercially available two-dimensional FEM program, Plaxis BV (1998), which automatically generates an element mesh. Thus the elements, nodes and Gauss point numbers varied slightly in each case.

The soil was assumed to be an elastic perfectly plastic material, obeying the Mohr–Coulomb failure criterion. The values of the geotechnical properties used for the FEM analysis are listed in Table 1, which were selected according to rock classification (Japan Society of Civil Engineers 1989), based on the assumption that the Ryukyu calcareous sediments can be classified as being between the CL class and D class. Such calcareous sediments are highly porous, with a reported coefficient of permeability in the range of $10^{-9}–10^{-8}$ cm/sec. Thus the drain behavior is expected in the field situation and the drained effective stress analysis was thus adopted in this study. The values of cohesion ($c$) and Young’s modulus ($E$) were chosen as an upper limit of the range previously measured. For the internal friction angle ($\phi$), an average value was used. The dilatancy angle ($\psi$) was assumed to be either 0 or 5°, to examine the influence of the value of $\psi$ on the calculated results. As will be described later on, as the value of $\psi$ did not affect the results prior to yielding in the scope of the present study, the dilatancy angle ($\psi$) was taken as being zero. The tensile strength ($\sigma_t$) was assumed to be 0.5 times $c$. The values of material properties of the footing were commonly used values within the linear elastic stress–strain range of standard concrete.

Numerical procedures were as follows. Having performed an initial stress analysis, excavations were performed at the nodes to form prescribed voids in the ground, after which a uniformly distributed strip load ($q$) with a load increment was applied on the footing until the ground reached the failure state. The load increment is automatically decided with trial calculation in the program. The calculation was terminated when the load increment automatically specified by the program was negative twice in succession.

**Evaluation Method for Effect of a Single Void**

Azam et al. (1997) used the reduction factor ($R$) to express the effect of a single void on the bearing capacity, which was defined by $R=q'_b/q_u$, where $q_u$ and $q'_b$=bearing capacity of the strip footing on the ground with a void and without a void, respectively. Similarly, the effect of a void is also evaluated in this paper by $R$,

<table>
<thead>
<tr>
<th>Physical property</th>
<th>Soil</th>
<th>Spread footing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Submerged unit weight $\gamma'$ (MN/m$^3$)</td>
<td>$9.0 \times 10^{-3}$</td>
<td>$1.4 \times 10^{-2}$</td>
</tr>
<tr>
<td>Poisson’s ratio $\nu$</td>
<td>0.3</td>
<td>0.1</td>
</tr>
<tr>
<td>Young’s modulus $E$ (MN/m$^2$)</td>
<td>$4.9 \times 10^2$</td>
<td>$3.0 \times 10^4$</td>
</tr>
<tr>
<td>Cohesion $c$ (MN/m$^2$)</td>
<td>0.98</td>
<td>—</td>
</tr>
<tr>
<td>Internal friction angle $\phi$ (deg)</td>
<td>26.5</td>
<td>—</td>
</tr>
<tr>
<td>Dilatancy angle $\psi$ (deg)</td>
<td>0</td>
<td>—</td>
</tr>
<tr>
<td>Tensile strength $\sigma_t$ (MN/m$^2$)</td>
<td>0.49</td>
<td>—</td>
</tr>
</tbody>
</table>
not in terms of bearing capacity but in terms of yielding pressure. Namely, the definition of $R$ adopted in this paper is

$$R = q_y / q_y'$$

(1)

where $q_y$ and $q_y'$ = yielding pressure of the strip footing on the ground with a void and without a void, respectively. The value of $R$ is an indicator of the degree of effect of a void on the yielding pressure. Namely, $R$ equals unity means that a void has no influence on the yielding pressure of the footing, and that the yielding pressure is the same as that for the ground without a void. Fig. 2 shows the relationship between loading pressure ($q$) and footing settlement ($S$) on a double logarithmic plot for the cases of without a void and with a void. In Fig. 2, the ultimate bearing capacity without a void, calculated by Terzaghi’s bearing capacity formula, is also indicated. The load pressure–settlement curve appears to converge at the value of Terzaghi’s ultimate bearing capacity at the point the settlement becomes as large as 1,000 mm, which corresponds to 0.5 times the footing width ($B$). However, the calculated results in these large deformation regions tend to fluctuate, thus the value of the $R$ factor cannot be accurately defined. Alternatively, for more consistent and accurate evaluation of $R$, a deflection point clearly observed in the load pressure–settlement curves in Fig. 2 was selected as the yielding pressure, which was used for the evaluation of $R$ in this study. It may be appropriate to evaluate the yielding pressure defined previously in relation to the allowable bearing capacity. In practice, a pressure at allowance bearing capacity. In practice, a pressure at

$$B = 0 \text{ mm}$$

is sometimes regarded as a limiting pressure when the failure pressure requires excessive settlement, as can be seen in Fig. 2 (Japanese Geotechnical Society 2004). Following this practice, the safety factor (yielding pressure/limiting pressure at $S/B=0.1$) was calculated and found to be in the range of 1.2–1.9 in this study.

Fig. 3 shows the progress of the failure zone for the case without voids for various footing settlements ($S$) in the range from 0.25 times the yield settlement ($S_y$) to $S_y'$. The vertical and horizontal scales in Fig. 3 are normalized by the footing width ($B$). The open squares in Fig. 3 indicate compressive failure, whereas the shaded circles indicate tensile failure at Gauss points. Compressive failure means that stresses at the Gauss points lie on the line of Coulomb’s failure criterion, whereas the tensile failure is a stress state where the minor principal stress ($\sigma_3$) reaches the tensile strength ($\sigma_t$). Fig. 4 shows stress paths of Point A and B shown in Fig. 3, together with the line of Coulomb’s failure criterion, in a major and minor principal stress diagram. In the figure, the positive stress components indicate compressive stress, whereas the negative stress components mean tensile stress. The compressive failure starts from the both edges of the footing when the footing settlement becomes 0.5 $S_y'$ as is seen in Fig. 3(b). The failure zone rapidly extends after 0.5 $S_y'$. When the footing settlement value becomes 0.8 $S_y'$ as is shown in Fig. 3(c), the zone of compressive failure further extends to the depth equal to footing width ($B$) and tensile failure appears at Point B. Evidently, $\sigma_3$ at Point B reaches $\sigma_t$ as is seen in Fig. 4. When the
footing pressure reaches the yielding pressure \( q_y \) shown in Fig. 3(d), the zone of the compressive failure forms an active wedge beneath the footing commonly assumed in the general failure mode. Correspondingly, the stress state at Point A is on the line of Coulomb’s failure criterion, as is shown in Fig. 4. It is considered, therefore, that if the yielding pressure is taken as a reference pressure, the failure zone may be extended.

As previous researches (Baus and Wang 1983; Al-Tabbaa et al. 1989) confirmed that the shape of a void has a negligible influence on the bearing capacity characteristic, a void having a square shape was used throughout this study. Fig. 5 shows a schematic view of the footing and single square void system. Symbols \( B \) and \( W \), respectively, designate the footing width and the void width, while symbols \( X \) and \( Y \), respectively, indicate the horizontal distance from the void center to the centerline of the footing, and the vertical distance from the ground surface to the center of the void. The values of \( X \) and \( Y \) were varied in order to examine the effect of the location of the void relative to the footing on the yielding pressure. The void has a square shape of \( W/B = 1.0 \) (\( W = 2 \) m and \( B = 2 \) m). The void eccentricity \( X \) was changed from 0 to 10 m, and \( Y \) from 1 to 11 m at 2 m intervals. In total 36 cases were analyzed.

To examine the consistency of the present study with the previous studies (Badie and Wang 1984; Al-Tabbaa et al. 1989), comparisons were made and the results are given in Fig. 6(a) with respect to the value of the reduction factor \( R \) defined by the Eq. (1) versus the void eccentricity/footing width ratio, \( X/B \), and in Fig. 6(b) with respect to the value of \( R \) versus the void depth/footing width ratio, \( Y/B \). Both the present study and the previous studies show the same general trend, i.e., that the value of the reduction factor increases with increasing \( X/B \) and \( Y/B \), regard-
less of the shape and size of the void. The value of \( R \) in the present study was consistently above 0.8, and when the value of \( Y/B \) was 3.0 or larger, the value of \( X/B \) was 2.0 or larger. Further, the influence of the void appears to be negligible with an \( R \) value of 0.9 or larger when either \( Y/B \) or \( X/B \) exceeds 5.0. It may therefore be considered that the yielding pressure is substantially unaffected by the existence of a single void if \( Y \) is over 3.0 times \( B \), or \( X \) is over 2.0 times \( B \). These findings in the present study agree with the previous results, as is shown in Fig. 6.

Fig. 7 shows the effect of the soil parameter on the value of \( R \) in terms of a nondimensional parameter \( \lambda_{\phi} = \gamma B \tan \phi/c \) for the case of \( W/B = 1.0 \), \( X/B = 0.0 \), and \( Y/B = 1.5 \). In the Fig. 7, the solid lines refer to cases of the \( \gamma B/c \) constant, whereas the dotted lines indicate the cases of the tan \( \phi \) constant. It is known that a change in the tan \( \phi \) value significantly affects the value of \( R \); therefore, the value of \( R \) decreases almost linearly with the increase in \( \lambda_{\phi} \) value on the logarithmic scale. The influence of the dilatancy angle (\( \psi \)) on the calculated results was found to be very minor prior to the point of yielding. In fact, the value of \( \psi \) in the range of zero to five had virtually no effect on the yielding pressure. As the present study focused on the yielding pressure of limited plastic strain development, the dilatancy angle (\( \psi \)) was assumed to be zero.

The following discusses how a single void influences the transmission of the footing pressure into ground material under the elastic condition and the effect on the failure mechanism in the plastic state. In Fig. 8(a), shown is a pressure bulb superimposed over the ground space normalized by \( B \), which is based a uniform surface pressure exerted on the strip footing (Taylor 1963). Contour lines indicate the ratios of a vertical stress (\( \sigma_z \)) to the loading pressure (\( q \)), suggesting that only 20% of \( q \) would be transmitted at the depth of \( Y/B = 3.0 \). Fig. 8(b) presents the failure mechanism of strip footing based on plastic theory for the case where the footing base is perfectly rough (Terzaghi and Peck 1967). The void at \( X/B = 2.0 \) corresponds to the case where the void is located near the middle of the passive zone. These simplified theories support the results shown in Fig. 6.

Fig. 9 shows the extent of the failure zone for the case of a void at \( Y/B = 1.5 \) with varying \( X/B \) values. For the case of \( X/B = 0.0 \) as shown in Fig. 9(a), the failure zone does not form the active wedge as was seen in Fig. 3(d) for the case of no voids. The compressive failure extends vertically along the lines connecting the edge of the footing and the upper corners of the void. This failure pattern was also observed in the model tests performed by Al-Tabbaa et al. (1989). On the contrary, tensile failure occurs both in the upper and bottom parts of the void, as if a built-in beam were subjected to bending. The mobilized resistance zone, which opposes the loading pressure, is confined in a narrow area between the footing and the void, resulting in the small value of the reduction factor (\( R \)) of 0.360. For the case of \( X/B \) as 1.0 shown in Fig. 9(b), a tilting failure zone develops from the both edges of the footing to the nearest corner of the void. This result is also consistent with the model test results obtained by Al-Tabbaa et al. (1989). From Fig. 9(b), it can be seen that compressive failure dominates in the failure zone on the left, whereas the failure zone on the right includes both compressive and tensile failures, indicating that the major resistance against loading pressure is mobilized along the left failure zone. The obtained value of the \( R \) factor increases to 0.704. For the cases where \( X/B \) is above 3.0, as shown in Figs. 9(c and d), formation of the active wedge beneath the footing is clearly seen, which is similar to the case without a void. Correspondingly, the value of \( R \) is close to unity.

Combining the results of Figs. 6(a and b), the values of the reduction factor (\( R \)) were plotted in a three-dimensional diagram, shown in Fig. 10(a). For a better understanding of the same data, Fig. 10(b) shows the effect of a void on \( q_y \), where the contours of \( R \) are given in normalized ground space. From Fig. 10(b), it can be seen that the contours are parabolic, and that \( Y \) has a more marked effect than \( X \), if the distance from the center of the footing

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**Fig. 8.** Point at which influence of single void disappears: (a) pressure bulb resulting from uniformly distributed surface pressure on strip footing; (b) failure mechanism of strip footing based on plastic theory

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**Fig. 7.** Effect of soil parameter on reduction factor (\( R \))
base to the center of the square void is the same. If the location of the void was identified by site investigation, Fig. 10 readily provides the degree of reduction of $q_y$ of strip footing resting on the ground surface due to the void. For example, if a void exists outside the contour line of $R=0.6$, it is estimated that the $q_y$ value of strip footing is greater than 60% the nonvoid yielding pressure value ($q'_y$). The effect of a void vanishes when the void exists under the contour line of $R=1.0$, which is called the critical location. Badie and Wang (1984) also pointed out the existence of a critical location, which is shown in Fig. 10(b). Their line is slightly deeper and wider. It should be noted here that their results were obtained for voids larger than those in the present study, and also that the shape of the void assumed by Badie and Wang was a circle, as opposed to a square in the present study. These factors may be responsible for the deviation in the results.

**Effect of Multiple Voids on Bearing Capacity Characteristics**

It would be convenient and practical if the reduction factor ($R$) for the yielding pressure ($q_y$) of strip footing on the ground with multiple voids could be estimated based on the reduction factor ($R$) for a single void, as described above, as given by

$$GR = R_A \times R_B \times \cdots \times R_N$$

where $GR$ is the global reduction factor, which is assumed to be given by multiplying the reduction factors for single voids, $A$, $B$, ..., $N$. If this equation is practically valid, Fig. 10(b) can be utilized for calculating the global reduction factor ($GR$). For this purpose, a second series of two-dimensional FEM analyses were carried out.

**Fig. 10.** Degree of reduction of yielding pressure ($q_y$) of strip footing resting on ground surface due to void: (a) variation of reduction factor ($R$) in three-dimensional diagram; (b) visual understanding of effect of void in contours.

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The degree of accuracy of Eq. (2) may be judged by the following index $\beta$:

$$\beta = \frac{GR_{FEM}}{GR}$$  \hspace{1cm} (3)

where $GR_{FEM}$ denotes GR obtained from FEM. If the value of $\beta$ is close to unity, Eq. (2) may be of practical use.

The effect of a two-void configuration (Void A and Void B) was examined first. Fig. 11 shows four two-void configurations: symmetrical, parallel, serial, and offset. The shape and size of each void is the same as in the single void case, namely the square void of $W/B=1.0$ [2 m width ($W$) and 2 m footing width ($B$)]. Symbols $X_1$ and $Y_1$ designate horizontal and vertical distances between two void centers, respectively. Fig. 11(a) shows the symmetrical configuration, for which the void eccentricity ($X$) was varied from 1.5 to 5.5 m at 1 m intervals, and the void depth ($Y$) was changed from 2 to 6 m at 1 m intervals, totaling 25 cases. Fig. 11(b) presents the parallel configuration. The vertical distance between two void centers ($Y_1$) was zero. Fig. 11(c) illustrates the serial configuration in the vertical direction, where the horizontal distance between two voids centers ($X_1$) was zero. Fig. 11(d) is the offset configuration, i.e., neither symmetrical nor parallel. For all the cases, except for the symmetrical configuration, the location of void A was $X_1/B=0.0$ and $Y_1/B=1.5$, and the location of Void B was varied. For the parallel configuration, the values of $X_1$ were 4 and 6 m. For the serial configuration, the values of $Y_1$ were 4 and 6 m. For the offset configuration, the values for $X_1$ and $Y_1$ were 4 and 6 m.

Two criteria were adopted for examining the validity of Eq. (2). One was with respect to the yielding pressure ($q_y$). The other was with respect to the loading pressure ($q$) at which the footing settlement ($S$) reached 25 mm, corresponding to a half to a third of the yield settlement value ($S_y$).

The values of $\beta$ are plotted against $GR$ defined in Eq. (2) with respect to the 25 mm footing settlement in Fig. 12(a) and with respect to the yielding pressure ($q_y$) in Fig. 12(b), respectively. The value of $\beta$ for the symmetrical configuration gradually decreases with increase in the GR value from 1.43 at GR=0.2 to 1.0 at GR=0.95, whereas the $\beta$ values for the other configurations lie in the narrow zone of $\beta=1.15–1.05$ and GR=0.3 to 0.35, as is seen in Fig. 12(b). More importantly, all the $\beta$ values are larger than unity, suggesting that Eq. (2) enables prediction with a comfortable margin of safety to be made. For the cases where the criterion using 25 mm footing settlement was adopted, the $\beta$ values are scattered over both the safe and unsafe sides, but lie in a very narrow range of 1.03 to 0.97, implying that the Eq. (2) is practically valid.

Fig. 13 compares the failure zones formed at $q_y$ for various void configurations. Except for the symmetrical configuration shown in Fig. 13(a), the failure zone in the other configurations develops toward the Void A, which is closest to the footing, suggesting that it is unaffected by void B. This failure pattern is very similar to the bending failure mode observed in Fig. 9(a) for a single void, and consequently the calculated value of $GR_{FEM}$ is 0.360, which is identical to the value for the case of a single void. For the symmetric configuration of the two voids shown in Fig. 13(a), the failure extends toward both voids, and the calculated $GR_{FEM}$ is 0.534, larger than in other cases. One possible reason for this is that the resistance in the zone beneath the footing is

![Fig. 12. Relationship between value of $\beta$ and $GR$ defined in Eq. (2) for: (a) 25 mm footing settlement ($S$); (b) yielding pressure ($q_y$).](image)

$$GR = R_A \times R_B$$
more effective in this particular configuration. This result is consistent with our theory that the critical failure mode is always formed through the weakest zone. It is also noted that the overall failure mode very similarly resembles the one with a single void, and that the parallel configuration exhibits the most tensile failure points of the three configurations.

The next question was whether Eq. (2) still holds when the number of voids is more than two. Figs. 14(a-d) show the failure mode for three different configurations of three voids, and one configuration of four voids, together with the values of \( GR_{\text{FEM}} \). Once again the results reveal that the critical failure mode is always formed through the weakest zone with the same value of \( GR_{\text{FEM}} \), regardless the number of voids and the void configuration. The extent of failure is also similar to the case of a single void, that is to say, the failure zone develops toward the nearest void, A. The values of \( \beta \) are 1.17 and 1.30 for three voids, and 1.4 for four voids, all with an adequate margin of safety. Thus, as long as the voids are of uniform shape and size, proposed Eq. (2) holds, and is therefore useful in a practical situation. However, it is noted that the accuracy of the proposed Eq. (2) appears to slightly deteriorate with an increase in the number of voids.

**Example for Bearing Capacity Calculation in Design**

In practice, a much more random configuration of voids is expected. Fig. 15 shows an example of how such situation is handled, and is based on voids of uniform shape and size. Suppose that the four voids, A, B, C, and D, were identified during site investigation prior to the start of construction, as is shown in Fig. 15(a). The failure zone at the yielding pressure obtained by FEM is shown in Fig. 15(a), and the calculated \( GR_{\text{FEM}} \) is 0.642, which is slightly smaller than the reduction factor of 0.678 for the nearest single Void D. Thus the calculated \( GR_{\text{FEM}} \) is slightly on the unsafe side. The reason for this is that, as is seen in Fig. 15(a), the failure zone extends further down, to void C. If the \( GR_{\text{FEM}} \) is calculated by using the reduction factor for single Voids C and D, the accuracy (\( \beta \)) can be improved from 1.16 to 1.06. Therefore, the global reduction factor (GR) may not be simply calculated from the multiplication of \( R \) for all the voids. One practical design option that could be adopted is, first, to calculate the void reduction factor for each single void and identify the voids with an \( R \) value smaller than 0.9; then multiply the lowest two \( R \) values to calculate the global reduction factor. The global reduction factor thus obtained is found to be always on the safe side. It is sometimes the case that unexpected additional voids are found during the course of construction, such as the void E, as shown in Fig. 15(b). Engineers on site may be required to make a quick evaluation of the stability of such voids and to decide whether countermeasures are needed, without performing detailed FEM analysis. In these circumstances, Eq. (2) together with Fig. 10(b) may be conveniently used. The calculated reduction factor for Void E is 0.196 and the value of \( GR_{\text{FEM}} \) is 0.182. To maintain the \( GR_{\text{FEM}} \) at the original design value, therefore, voids such as these...
could be filled with a grout material, which would increase their
strength to a sufficient level.

Summary and Conclusions

The effect of single and multiple square voids in the ground on
the yielding pressure for a spread footing was investigated nu-
merically, using two-dimensional FEM analysis. The results were
quantitatively evaluated using the reduction factor of the yielding
pressure. Based on the results of the present study, the following
conclusions are drawn.

1. If a single void exists in the ground, the yielding pressure of
the surface footing decreases, which is affected by two pa-
rameters indicating the location of the void; void depth/
footing width, and void eccentricity/footing width. The fail-
ure zone extends from the edge of the footing toward the
nearest corners of the void, without forming an active wedge
beneath the footing. A diagram illustrating the reduction in
of the yielding pressure is provided. There exists a parabolic
critical line beyond which the effect of a single void on
the yielding pressure of the footing vanishes, where full forma-
tion of an active wedge beneath the footing is observed.

2. The previous results of the present study for a single void
agree well with the previous experimental and numerical
studies, both in terms of failure pattern and reduction factor.

3. If two voids with the same size for various configurations
exist in the ground, there is a strong tendency for a failure
zone to develop near the nearest void, and thus the reduction
in yielding pressure can be easily evaluated with reasonable
accuracy by multiplying the reduction factors for each single
void. For cases of more than two voids of the same size, the
reduction in the yielding pressure is practically estimated by
multiplying the reduction factors for the two voids closest to
the footing among the voids having a reduction factor
smaller than 0.9.

Notation

The following symbols are used in this paper:

- \( B \) = footing width (m);
- \( c \) = cohesion (MN/m²);
- \( E \) = Young’s modulus (MN/m²);
- \( \text{GR} \) = global reduction factor calculated from Eq. (2);
- \( \text{GR}_{\text{FEM}} \) = global reduction factor obtained from FEM;
- \( q \) = loading pressure (MN/m²);
- \( q_y \) = yielding pressure of footing on ground with void
  (MN/m²);
- \( q'_y \) = yielding pressure of footing on ground without
  void (MN/m²);
- \( R \) = reduction factor;
- \( R_A \) = reduction factor for single void A;
- \( R_B \) = reduction factor for single void B;
- \( S \) = footing settlement (mm);
- \( S_y \) = yield settlement of footing on ground with void
  (mm);
- \( S'_y \) = yield settlement of footing on ground without
  void (mm);
- \( W \) = void width (m);
- \( X \) = void eccentricity (m);
- \( X_1 \) = horizontal distance between two voids centers
  (m);
- \( Y \) = void depth (m);
- \( Y_1 \) = vertical distance between two voids centers (m);
- \( \beta \) = degree of accuracy using equation (2);
- \( \gamma' \) = submerged unit weight (MN/m³);
- \( \lambda_{g0} \) = non-dimensional soil parameter;
- \( \nu \) = Poisson’s ratio;
- \( \sigma_t \) = tensile strength (MN/m²);
- \( \sigma_y \) = vertical stress (MN/m²);
- \( \sigma_1 \) = major principal stress (MN/m²);
- \( \sigma_2 \) = minor principal stress (MN/m²);
- \( \phi \) = internal friction angle (deg);
- \( \psi \) = dilatancy angle (deg).

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