Optimal seat and suspension design for a quarter car with driver model using genetic algorithms

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Abstract

This paper presents an optimization of a four-degrees-of-freedom quarter car seat and suspension system using genetic algorithms to determine a set of parameters to achieve the best performance of the driver. Since the health of the driver is as important as the stability of the car, the desired objective is proposed as the minimization of a multiobjective function formed by the combination of not only suspension deflection and tire deflection but also the head acceleration and crest factor (CF), which is not practiced as usual by the designers. The optimization results are compared through step and frequency responses of the seat and suspension system for the optimum and currently used suspension systems. Comparatively better results are obtained from the optimized system in terms of resonance peaks, CF, and vibration dose value. The concept and the ideas set forth in this work are directly applicable to both the car suspension and seat design in industry.

Keywords: Quarter car; Suspension; Optimal seat design; Car-driver model

1. Introduction

Exposure to whole body vibration (WBV) associated with a prolonged seating is an important risk factor for low back pain (LBP) among drivers (Wilder, 1993; Pope et al., 1998; Paddan and Griffin, 1998; Bovenzi and Hulshof, 1999; Griffin, 1998; Johnson and Neve, 2001). Both vehicle suspension system and driver seat cushion designs have attracted significant interest over the last several decades with a significant effort being directed towards their improvements. Vibration attenuation through the suspension and seat will not only provide riding comfort but also reduce the risk of LBP due to driving.

One of the early studies on the biomechanics of seated drivers subject to vibration was realized by Suggs et al. (1969), where the human body was modeled as a damped spring-mass system to build a standardized vehicle seat testing procedure. Mukhsian and Nash (1974) and Pope et al. (1987) were investigated the response of seated humans to sinusoidal vibration and impact. A detailed experimental work on translational seat vibration was performed by Griffin et al. (1982) to determine the effects of level, frequency and direction of the seat vibration. Dynamic response of a seated subject was investigated in various aspects, for example, the effect of various cushions (Pope et al., 1989), the effect of vibration frequency and posture (Zimmermann and Cook, 1997; Wilder et al., 1994), and the effect of backrest (Cho and Yoon, 2001; Lewis and Griffin, 1996). Wan and Schimmels (1997) established a seated human body model to design an optimal seat suspension for isolation of the vertical WBV based on the simulated subjective response. On the other end of the spectrum, the effect of spinal forces due to WBV (Fritz, 1997; Kumar et al., 1999; Verver et al., 2003) and sitting biomechanics (Harrison et al., 2000) were considered in some other studies.

Most of the studies on this subject treat the seat and suspension designs separately. This study, however, integrates both topics into a single problem through a multiobjective optimization. The quarter car and the seat with driver’s body are simply modeled together as a
four-degrees-of-freedom (dof) damped spring-mass system to analyze the dynamic response of the human body and design the seat and suspension system optimally to get best performance of the driver subjected to WBV. A number of criteria must be taken into account to meet certain stability and comfort considerations (Deb and Saxena, 1997). Therefore, the objective function combining the head acceleration (HA), its crest factor (CF), the suspension deflection (SD), and the tire deflection (TD) are used to provide not only the stability of the car but also the comfort of the driver. Optimizations are performed by using genetic algorithms (GA). The results are presented in both tabular and graphical means, and compared among the individual objective functions in terms of CF and vibration dose value (VDV) (Smeathers and Helliwell, 1993). It is shown that comparatively better results were obtained in the case of the multiobjective function.

2. Model development

Construction of human body models for the simulation of vibration exposure characteristics is not a unique process (Tregoubov, 2000). The same exact data can be produced from different models and several sets of model parameters in accordance with the experimental data. In the literature, one may find a number of different models ranging from several dof (Wei and Griffin, 1998) to comparatively higher dof (Arirouche and Ider, 1988; Kim et al., 2003; Qassem, 1996) systems with linear (Alkhatib et al., 2004) or non-linear (Rakheja et al., 1994; Wan and Schimmels, 2003) parameters. Since the aim of this study is to propose an optimization method to the suspension and seat designers and to attract their attention to important metrics of such a design, a simplest form of human body model has been chosen to study with.

In particular, a four-dof dynamic model for a car suspension system (Fig. 1) is established to optimize the dynamic response of the car and driver system in a way to minimize the force transmitted to the lower back and to minimize the acceleration of head and upper body. In the model, it is assumed that the tire is always in contact with the road during the motion, and it is treated as a linear spring with a constant of \( k_y \). The constants \( m_t \) and \( m_p \) are the sprung and unsprung mass of the vehicle. \( c_s \) and \( k_s \) are the constants representing, respectively, the damping and stiffness coefficients of the suspension. Elastic properties of the cushion are also modeled as a spring and a dashpot with respective constants of \( k_c \) and \( c_c \). The driver is represented by a two dof lumped mass system. The upper body is also connected to the lower part through a spring and a dashpot combination with constants, respectively, \( k_t \) and \( c_t \). Furthermore, it is assumed that the system is free to move in the vertical direction only, and the motion in other directions is considerably small and hence neglected. Finally, \( z_t \) and \( z_p \) are the sprung and unsprung mass displacements measured from static equilibrium positions while \( z_r \) represents the road profile. The variables \( z_t \) and \( z_p \) represent the possible displacements of the thorax and pelvis from their static equilibrium positions.

The dynamic equations of motion for the system can be described by the following differential equations from their static equilibrium positions:

\[
\begin{align*}
    m_t \ddot{z}_t &= -c_s(z_t - \dot{z}_p) - k_s(z_t - z_p), \\
    m_p \ddot{z}_p &= c_s(z_t - \dot{z}_p) + k_t(z_t - z_p) - c_c(z_p - \dot{z}_s) - k_c(z_p - z_s), \\
    m_s \ddot{z}_s &= c_c(z_p - \dot{z}_s) + k_c(z_p - z_s) - c_s(z_s - \dot{z}_u) - k_s(z_s - z_u), \\
    m_u \ddot{z}_u &= k_s(z_s - z_u) + c_s(z_s - \dot{z}_u) - k_y(z_u - z_t).
\end{align*}
\]

![Quarter car model with driver.](image)

The dynamic model of the system is a coupled linear differential equation set in four system variables of \( z_t, z_p, z_s \) and \( z_u \) for a given road profile \( z_r \). The mass of driver is taken as \( m = 65 \text{ kg} \), while the mass of upper body and head is taken as \( m_t = 2 \text{ m}/7 \), and the mass of lower body and seat is taken as \( m_p = 5 \text{ m}/7 \) as apparent masses (Tewari and Prasad, 1999). The quantities used in the simulations for sprung and unsprung masses are as follows: \( m_t = 240 \text{ kg} \), \( m_u = 36 \text{ kg} \), \( c_s = 980 \text{ N/s/m} \), \( k_s = 16000 \text{ N/m} \), \( k_y = 160000 \text{ N/m} \) (Gillespie, 1992). The damping and stiffness constants for thorax are given as \( c_t = 1360 \text{ N/s/m} \) and \( k_t = 45005.3 \text{ N/m} \), respectively.
aspects of the dynamic system. The sprung mass acceleration (SMA), the SD and the TD are the three commonly used performance indices for the suspension systems in the literature (Li and Kuo, 2000). Since the acceleration and force transmitted to the upper body and head are the most important factors affecting drivers’ health and comfort (Wilder, 1993; Pope et al., 1998; Bovenzi and Hulshof, 1999; Griffin, 1998; Deb and Saxena, 1997), the HA and CF are used to form the multi-objective function besides the SD and the TD. Since the HA is related with the transmitted force with a proportionality constant of $m_t$ (see Eq. (1)), the HA is included in the objective function. The mixed objective function is then composed according to global criterion method (Rao, 1996) in which the relative deviations of the objective functions from their feasible ideal solutions are minimized as

$$J_{\text{mixed}} = \sum_{i=1}^{4} \left( \frac{J_i - J_i^0}{J_i^0} \right)^2 \text{ for } i = \text{HA, CF, SD, and TD},$$

where the individual objectives can be expressed as follows:

$$J_{\text{HA}} = \frac{1}{2} \int_0^\infty \left( \ddot{z}_t \right)^2 \, dt$$

$$J_{\text{CF}} = \frac{z_{\text{max}}}{z_{\text{rms}}},$$

$$J_{\text{SD}} = \frac{1}{2} \int_0^\infty (z_t - z_u)^2 \, dt,$$

$$J_{\text{TD}} = \frac{1}{2} \int_0^\infty (z_t - z_p)^2 \, dt,$$

and $J_i^0$ represents the $i$th individual objective function evaluation for its ideal solution. By dividing each term in Eq. (5) to its ideal solution, non-dimensional expressions are obtained to ensure equal units for each objective function. However, equal importance is given to each of the objective function described earlier.

The objective function is obviously multimodal (Deb and Saxena, 1997) and hence is minimized using GA. GA are search algorithms based on the principles of genetics and the theory of evolution. Therefore, much of the terminology used in the GA literature is adopted by engineers from biology such as genes (bits), chromosomes (bit strings), and population of individual sets (structures) (Michalewicz, 1996).

Since GAs require only function evaluations, without need for computationally expensive and often difficult to determine gradient information, they find a wide range of applications in optimization problems that can possess mixed continuous and/or discrete variables, as well as discontinuous and non-convex objective functions. Since GAs search from a distributed population of points, they

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### Table 1

<table>
<thead>
<tr>
<th>Method</th>
<th>$k_c$</th>
<th>$c_c$</th>
<th>$k_s$</th>
<th>$c_s$</th>
<th>CF</th>
<th>SD</th>
<th>TD</th>
<th>HA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Classic 1</td>
<td>2500</td>
<td>131.59</td>
<td>16000</td>
<td>980</td>
<td>2.9</td>
<td>0.65</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Classic 2</td>
<td>20000</td>
<td>1649.03</td>
<td>16000</td>
<td>980</td>
<td>8.9</td>
<td>1.25</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SMA</td>
<td>3375</td>
<td>1361</td>
<td>8045.0</td>
<td>517.3</td>
<td>7.9</td>
<td>0.64</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CF</td>
<td>12786.2</td>
<td>233.2</td>
<td>21876.6</td>
<td>1155.3</td>
<td>3.5</td>
<td>0.40</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SD</td>
<td>3947.5</td>
<td>574.8</td>
<td>23950.7</td>
<td>1450</td>
<td>2.8</td>
<td>0.90</td>
<td></td>
<td></td>
</tr>
<tr>
<td>TD</td>
<td>4917</td>
<td>1590.8</td>
<td>15836.1</td>
<td>1467.8</td>
<td>3.6</td>
<td>0.73</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mixed objective</td>
<td>2715.9</td>
<td>216.8</td>
<td>9886.9</td>
<td>851.9</td>
<td>3.3</td>
<td>0.65</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(Fairley and Griffin, 1989; Rosen and Arcan, 2003). The damping and stiffness coefficients for the cushion material is, respectively, given in a range of $c_c = [131.59, 1649.03]$ Ns/m and $k_c = [2500, 20000]$ N/m (Wan and Schimmels, 1997). The height of the step is specified as $z_s = 0.02$ m for a first trial.

Step responses of the system for the particular set of system parameters listed above are illustrated in Fig. 2 for both lower and upper bounds of the cushion material used in the literature. Input step function is also shown in the figure. Although the overshoot in the upper body displacement decreases with increasing cushion material constants, the CF based on HA increases drastically from 2.9 to 8.9, which is an extreme condition for the driver comfort (see Table 1). Therefore, the cushion material—as well as the suspension parameters—must be optimally adjusted so as to minimize the vibration transmitted to the driver.

### 3. Objective function definitions

One of the most significant problems in optimization is the choice of a proper cost function reflecting the various aspects of the dynamic system. The sprung mass acceleration (SMA), the SD and the TD are the three commonly used performance indices for the suspension systems in the literature (Li and Kuo, 2000). Since the acceleration and force transmitted to the upper body and head are the most important factors affecting drivers’ health and comfort (Wilder, 1993; Pope et al., 1998; Bovenzi and Hulshof, 1999; Griffin, 1998; Deb and Saxena, 1997), the HA and CF are used to form the multi-objective function besides the SD and the TD. Since the HA is related with the transmitted force with a proportionality constant of $m_t$ (see Eq. (1)), the HA is included in the objective function. The mixed objective function is then composed according to global criterion method (Rao, 1996) in which the relative deviations of the objective functions from their feasible ideal solutions are minimized as

$$J_{\text{mixed}} = \sum_{i=1}^{4} \left( \frac{J_i - J_i^0}{J_i^0} \right)^2 \text{ for } i = \text{HA, CF, SD, and TD},$$

where the individual objectives can be expressed as follows:

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$$J_{\text{CF}} = \frac{z_{\text{max}}}{z_{\text{rms}}},$$

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$$J_{\text{TD}} = \frac{1}{2} \int_0^\infty (z_t - z_p)^2 \, dt,$$

and $J_i^0$ represents the $i$th individual objective function evaluation for its ideal solution. By dividing each term in Eq. (5) to its ideal solution, non-dimensional expressions are obtained to ensure equal units for each objective function. However, equal importance is given to each of the objective function described earlier.

The objective function is obviously multimodal (Deb and Saxena, 1997) and hence is minimized using GA. GA are search algorithms based on the principles of genetics and the theory of evolution. Therefore, much of the terminology used in the GA literature is adopted by engineers from biology such as genes (bits), chromosomes (bit strings), and population of individual sets (structures) (Michalewicz, 1996).

Since GAs require only function evaluations, without need for computationally expensive and often difficult to determine gradient information, they find a wide range of applications in optimization problems that can possess mixed continuous and/or discrete variables, as well as discontinuous and non-convex objective functions. Since GAs search from a distributed population of points, they
have a good chance to catch global optimum, whereas other heuristic methods often move from a single point in searching optimum to the next using some transition rule to determine the next point. Therefore, in most cases, they tend to find local minimum that is closest to the initial point (Goldberg, 1989).

A detailed discussion and description of GAs, their underlying theory and structure will not be given here. The interested reader may instead refer to the many excellent texts which are available such as those by Michalewicz (1996) and Goldberg (1989). In the work presented within this paper, a GA implementing Goldberg’s algorithm in MATLAB® was used for optimizations. It uses fixed population size with string length of 30, a crossover probability, $P_c$, of 0.001, and a mutation probability rate, $P_m$, of 0.002.

In the GA, a population of points with each point representing a set of design parameters has to be formed in a given range. Adjusting this range is not a well-defined process. However, it is not greatly erroneous to estimate these parameters around the classic values within a certain range. If one considers that these commonly used classic values have resulted from decades of experience, these values are indeed the most logical region of the design space to begin and concentrate a search. Therefore, 50% lower and upper limits are utilized in this work, and the results are compared among themselves in both tabular and graphical manner. It is imperative here to note that this range should not exceed above a certain limit because the realization of the physical system may not be feasible for higher values of potential solutions.

The design parameters are the stiffness and damping coefficients of the cushion and car suspension. The upper and lower bounds for the cushion parameters are taken from Wan and Schimmels (1997) as $c_c = [131.59, 1649.03] \text{Ns/m}$ and $k_c = [2500, 20000] \text{N/m}$. The range for the suspension parameters are constructed as 50% below and above the classic values mentioned earlier, i.e., the ranges are taken as $c_s \in [0.50c_s, 1.50c_s]$ and $k_s \in [0.50k_s, 1.50k_s]$.

4. Simulation results

Optimizations are performed using the objective functions aforementioned and the results are tabulated in the table. Besides the spring and damping coefficients, the CF and the VDV are also included for the response to the prescribed step input for each case investigated.

As a comparison, the step responses, the resulting HAs, and the transmitted force between the pelvis and thorax of the system with both the optimized parameters and the classical one are depicted together in Figs. 3–5. In the simulations, a step height of 0.02 m is used as performed previously. Input step function is also shown in Fig. 3.

Maximum overshoot and settling time are the two of the most critical values in a design process because they are directly related with CF and VDV. In a step response such
as the one illustrated in Figs. 2 and 3, these values can easily be measured. Moreover, it is also possible to visualize how oscillatory the system in each case presented, which is directly related to the driver’s comfort. In each case described previously, the optimized system is less oscillatory and has a very low maximum percent overshoot and settling time as compared to the un-optimized case (Fig. 3).

Frequency response of the transmitted force between the pelvis and thorax is also drawn in Fig. 6 for low frequencies. It is clear from the response that a relatively better result is obtained in the transmitted force magnitude around the resonance peak.

5. Conclusion

In this work, a specified quarter car model with driver seat and driver involving four-dof system is used in an optimization problem to determine an optimum set of parameters to achieve the best performance of the system.

The model is assumed to have four masses attached with linear springs and dampers. It is also assumed that the system does not vibrate in lateral and longitudinal directions, only oscillates in vertical (vertical to road surface) direction. Furthermore, the tires are assumed not losing the contact with the road surface.

It is obvious from the response plots that optimal solutions are less oscillatory, which naturally brings stability and ride comfort. Moreover, these solutions have lower values of maximum over shoots and settling times, which result in lower CF and VDV.

In an optimum design of suspension and seat system, one must include not only stability but also health and comfort criteria in the objective. While the suspension and tire deflections provide the vehicle stability, the HA and CF provide driver’s comfort and health. Since the HA is directly related with the force transmitted between the pelvis and thorax, one of these two must be considered in the designs. CF or VDV might be included to ensure that the maximum acceleration levels are not reached.

As another criterion when deciding which optimum values to choose among these parameters, one may want to consider choosing the parameter set with lowest practical suspension spring to minimize the natural frequency of the sprung mass because road acceleration inputs increase in amplitude at higher frequencies. Therefore, the best isolation is achieved by keeping the natural frequency as low as possible.

References


Fig. 6. Frequency response of the transmitted force between the pelvis and thorax.


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