Computers & Industrial Engineering 95 (2016) 27–40

Contents lists available at ScienceDirect

Computers & Industrial Engineering

journal homepage: www.elsevier.com/locate/caie

Determination of optimal order-up to level quantities for dependent spare parts using data mining

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ARTICLE INFO

Article history:
Received 19 July 2014
Received in revised form 17 January 2016
Accepted 18 January 2016
Available online 16 February 2016

Keywords:
Data mining
Association rule
Dependency
Spare parts management

ABSTRACT

Consumable spare parts play an important role during regular and periodic maintenance activities depending on equipment criticality of a manufacturing firm. Usually, these spare parts are consumed together on a regular basis to different equipment with a high percentage of commonality. Complete or partial shutdown of maintenance activity may happen if there is shortages in any of these common parts and at the same time, maintaining a high inventory of these parts lead to more cost. If there is a shortage of a spare part, then there is a chance that other dependent spares may remain as idle inventory and incur the opportunity cost. In this paper, a noble approach is adopted to incorporate dependency of items in periodic review (T, S) policy to determine the optimal stock of dependent spare parts considering common cycle time and fill rate for each spares. Our results show that there is considerable reduction of stock level and total cost of inventory when associated spares are considered together for management of inventory instead of considering them individually. Finally, the developed model is applied to a case company.

1. Introduction

In today's competitive business environment, one of the important issues in a manufacturing firm is the management of spare parts. Consumable spare parts play an important role during regular and periodic maintenance activities depending upon equipment criticality. Many firms store thousands of spare parts in inventory that shares the major percentage of inventory cost. A large inventory of spare part results in high holding and obsolescence cost. On the other hand, low level of inventory in store causes severe production loss due to shortages.

During maintenance operations of equipment, many spare parts are consumed together depending upon their correlations among themselves. Quite often use of one spare part leads to the use of another spare part whereas sometimes it does not happen. For example, during the maintenance of a two wheeler when bearing races are replaced, there is a high chance that shock absorber spring also gets replaced. In this paper, we have termed these spares as associated spare parts. Among these spares, if one spare is out of stock, then there is maximum likelihood that the consumption of the other spare part also gets affected like fall in the demand rate or pile up of inventory in stock. In case of single item inventory control when there is a shortage or occurrence of stock out, only the effect of independent items is considered but in case of associated items, stock out of one item causes the excess of inventory holding of other related items. Usually, maintenance database of a company keeps a large number of past consumption records with respect to maintenance activities for various equipment. However, it is very difficult to use statistical analysis to estimate frequent consumption of spares in group and also the dependency. Even though over the years, many mathematical models have been developed in the literature for the management of independent spare parts, yet only a few of them have seen the light of actual application in the field due to its complex mathematical structure.

In recent times, data mining application has received considerable attention from various sectors like medical service, intrusion detection, and customer relationship management. Various methods of data mining such as association rule, classification, clustering, sequence mining, and regression are applied for the above areas (Han & Kamber, 2006). Association rule mining aims at understanding the relationships among items in transactions or market baskets (Agrawal, Imilienski, & Swami, 1993). But till date, no such application is seen in the areas of spare parts inventory management. This has motivated us to take up this new approach for the management of consumable dependent spare

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http://dx.doi.org/10.1016/j.cie.2016.01.024
0360-8352/

parts. In the current IT era, most of the firms have implemented ERP package and as a result all transaction data related to consumable spare parts are available with them. In this paper, data mining methodologies are used to estimate the dependency factor among the consumable spare parts and finally developed inventory control policy. Prior to association rule mining, frequent item sets are determined from database transactions. Frequent item set is the number of items consumed together in a particular period of time satisfying minimum threshold support value. Based on this frequent item set and association rule mining methodologies, a few authors have carried out studies related to item-item relationship or dependency and in the next section; a review of literature is provided.

2. Literature review

Currently, there are very few contributions of data mining are available in the domain of inventory management and more specifically on spare parts management. A brief review of literature related to our work on data mining applications in inventory management is discussed here. Brijs, Swinnen, Vanhoof, and Wets (1999) used association rules for item selection problem with the consideration of relationships among retail items to discover frequent itemsets and have determined the profitability per set of items by identifying the cross-sales effect of product items and used this information for better product selection. This model was extended by Brijs, Goethals, Swinnen, Vanhoof, and Wets (2000) to enable retailers to add category restrictions. Wong, Fu, and Wang (2005) followed the earlier work of Brijs et al. (1999) and proposed a method for actionable recommendations from itemset analysis and investigated an application of the concepts of association rules-maximal-profit item selection with cross-selling effect. Bala, Sural, and Banerjee (2010) proposed a model for finding purchase dependence association rules for retail products to take inventory replenishment decisions. As per their observation, in a multi-item retail inventory with a very large number of items, purchase dependence among the items is often observed and when there is stock-out of one item, it may result a non-purchase of another item. Yin, Kaku, Tang, and Zhu (2011) described the association rule mining in an inventory database. They have given brief explanations for frequent item/item sets, apriori algorithm, discovering association rules from frequent item sets, multi-dimensional association rules and association rules with time-window.

The basic method of data mining for finding association rules from frequent items as given in Yin et al. (2011) is explained here for the easy understanding of the reader. Suppose, we have a rule called $X \Rightarrow Y$ where X and Y are two item sets, then the rule confidence is given as follows:

$$\text{Confidence of rule } X \Rightarrow Y = Conf(X \Rightarrow Y) = \frac{|X \cup Y|}{|X|} = \frac{Sup(X \cup Y)}{Sup(X)}$$

where support of X is $Sup(X) = |X|/|D|$ and $|D|$ represents the whole data base records. A rule satisfying both threshold support and confidence values (which are identified by experts) is called strong or valid. The items inside these rules are called associated items. Liiv (2007) proposed an inventory classification method using visualization and data mining technique considering interdependency among the products. Association rule method of data mining is used for classification of inventories by Zhenyu, Yan, and Zhenying (2009). The authors used weighted association rule concept to build a new evaluation criterion based on weighted dollar consumption.

Adopting a classical approach, many researchers have proposed coordinated or joint replenishment policy for multi item inventory control. It is always beneficial to order items in group for better management of inventory control decisions rather than concentrating on individual item. Ramani and Krishnan (1985) suggested that it is necessary to group various spare parts into different categories, which are associated to a particular equipment or maintenance activity to apply different ordering policies (i.e. multi-item inventory control) according to the importance of each category. This leads to a lesser amount of effort for managing the stock of the spares that are falling in the same group and at the same time, number of spares becomes less that requires more management attention. Razi and Tarn (2003) developed another methodology of item grouping approach for estimating demands. These groups are based on annual demand units and lead time and a common group demand distribution was determined. They applied (T, S) periodic review policy to estimate the base stock level (S) with service level constraints.

Again, many researchers have studied coordinated or joint replenishment models for multi item inventory control systems. Among those, Balintfy (1964) proposed a (S, c, s) policy, which is also called can-order replenishment policy and is widely discussed in the literature. In this policy, each item can have a can-order level c in spite of their own must order level s. When the inventory position of item i reaches its must order level s_i, a replenishment order is placed. At the same time, any item j with inventory position at or below its can order level c_j is included in the order to make the order up-to level of S_i and S_j for item i and j respectively. Silver (1974) used this (S, c, s) policy for estimating order up-to level, can-order point and must order point using poisson demand and non-zero lead time. They proved that around 18.8% savings are resulted because of the coordinated model compared to independent model. Other researchers have also developed inventory control models for multiple items considering correlated demands. Liu and Yuan (2000) proposed coordinated replenishment model considering correlated demands for multi item inventory control. The authors utilized Markovian model for two item inventory system with correlated demands and estimated total cost per unit time. Finally, they concluded that cost saving increases when there is a decrease in demand covariance, and the total cost involved in coordinated policy with correlated demands is less than independent control.

Khouja and Goyal (2008) have provided an excellent review of literature on joint replenishment problems from 1989 to 2005. Fung, Ma, and Lau (2001) suggested (T, S) coordinated replenishment model for multi-item inventory systems considering compound Poisson demand, non-zero lead time and service level constraints. They made a comparison of their model with (S, c, s) model and concluded that (T, S) model gives better results than (S, c, s) model considering non-zero lead-times. An association clustering method was proposed by Tsai, Tsai, and Huang (2009) for can-order policies for joint replenishment problem, and the suggested model gives better result compared to several traditional replenishment models. A different approach of periodic review of multi-item inventory control was proposed by Nenes, Panagiotidou, and Tagaras (2010) by taking stochastic demand for fast and slow moving spares. They used Gamma and Poisson distribution for fast-moving and slow moving spares respectively to estimate the accurate base stock level with desired fill rate constraints.

Again, Pentico and Drake (2009) proposed a new methodology to determine EOQ with partial backordering considering fill rate and backordering percentages. Later Pentico, Drake, and Toews (2009) demonstrated similar work for EPQ model with partial backordering. Using the similar concept, Zhang, Kaku, and Xiao (2011) presented deterministic EOQ model for dependent products considering cross-selling effects between the items. They

Table 1
Spare parts consumption for different equipment.

MID	EQUIPMENT	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}
1	E4	1	1	1	1	1	1	1	1	1	1
2	E5	1	1	1	1	0	0	0	0	0	0
3	E5	0	0	0	0	1	0	0	1	1	0
4	E1	1	0	1	0	1	1	1	1	1	1
5	E1	1	1	1	1	1	1	1	1	1	1
6	E5	1	1	1	1	1	1	0	1	0	1
7	E2	1	1	1	1	1	0	1	0	1	1
8	E1	0	1	1	1	1	1	1	0	0	1
9	E3	1	1	1	1	1	1	0	0	1	1
10	E5	1	0	1	1	1	0	0	0	0	0
11	E2	0	1	0	0	0	0	0	0	0	1
12	E2	1	1	1	1	1	1	1	1	1	1
..	..	..	..	..	..	..	..	..	..	..	..
..	..	..	..	..	..	..	..	..	..	..	..
N	E1	1	0	1	1	0	1	1	0	1	1

Table 2
Yearly transactions of spare parts S_1 and S_2.

Year-I			Year-II			Year-III		
TID	S_1	S_2	TID	S_1	S_2	TID	S_1	S_2
1	√	×	8	√	√	18	×	√
2	√	√	9	×	√	19	√	×
3	√	√	10	√	√	20	√	√
4	×	√	11	√	√	21	√	√
5	√	√	12	√	×	22	√	√
6	√	√	13	√	×	23	√	√
7	×	√	14	×	√	24	×	√
–	–	–	15	√	√	25	√	√
–	–	–	16	√	√	26	√	√
–	–	–	17	√	√	–	–	–

considered two types of product like major and minor where partial backordering with lost sales is allowed for the major product and considering no stock out for minor products. Zhang (2012) extended the work proposed by Zhang et al. (2011) by incorporating multiple minor items, which are associated to each other. They used mixed integer non-linear programming approach to find out global optimum solution where the total inventory cost is minimum. They proposed joint replenishment model considering the correlations or association between the items with complete backordering. They used minor item ordering cycle time as an integer multiple of cycle time of major item and tried to optimize total cost function. In our present study, we have taken a cue from this work and extended the study to develop periodic review policy considering stochastic demand in the case of associated spare parts inventory control. Here, we have used a multi-item periodic review inventory control method to estimate the order up to level of associated spare parts. The main contribution of our study can be summarized as follows.

- The application of data mining tool in the presented approach for spare parts inventory control is a novel approach. None of the earlier spare parts management studies have considered part-part relationship or dependency in inventory modeling. Our study introduces spare parts dependency to inventory control modeling of total cost approach to determine optimal stock levels. The dependency factor has a direct effect on inventory holding and shortage cost of dependent spare parts.
- Our study has also contributed to the literature by revealing the fact that when dependency factor is high (major and minor spare parts are strongly dependent), dependent (minor) spare parts will bear more inventory carrying cost and less shortage cost if demand of major spare parts is satisfied with lesser fill rate.
- Further, our study has also revealed that total inventory cost of two spare parts increases with decrease of service level, i.e. fill rate and thereby contributed to the literature. The reason behind it is that by maintaining lower fill rate, major spare faces less inventory cost and very high shortage cost and this shortage increases the inventory holding of the minor or dependent spares due to dependency effect.

The rest of the paper is organized as follows. In Section 3, a simple procedure for estimating the dependency factor is described with an example for ease of understanding. Basic inventory control model for single spare part allowing shortage is also explained here so that one can easily visualize the problem when we consider multiple associated items. Section 4 describes the proposed inventory control model for two associated spares with a common review period and shortages. A solution procedure is described in Section 5, and a numerical example is discussed in Section 6 to illustrate the proposed model. In Section 7, we have presented an industrial case study where the proposed model was applied. Finally, conclusion and scope for further research are provided in Section 8.

3. Methodology

Here, first we have discussed the procedure to compute the dependency factor for associated spare parts considering historical consumption records. Next, the conventional periodic review policy (T, S) is mentioned considering shortages. This has been discussed to make the background work for our proposed model and for ease of understanding of the reader.

3.1. Extracting dependencies among spare parts

Several spare parts are consumed during each maintenance activity of equipment. In this case, the quantity of consumption of spare parts could be either single unit or more than one unit. For determining frequent spare parts group, we assume these transactions as binary, i.e. '1' means the part is consumed and '0' means it is not consumed. Table 1 shows different spare parts consumptions for different equipment with respect to maintenance transaction in a database (MID). Here, the whole data set of maintenance activities are transformed from quantitative to binary transactions for easiness of computing dependencies among the parts and is shown in Table 1.

First, a procedure is discussed here for computing the dependency factor for associated spare parts. For example, spare parts $S_1, S_2, S_3, \ldots, S_m$ are consumed for a specific maintenance activity of an equipment, and records are stored in a maintenance database as unique transactions called TIDs. Dependency factor of a group of spare parts can be computed as follows:

$$\alpha_{S_1S_2\ldots S_m} = \frac{1}{n}\sum_{i=1}^{n} Support(S_1S_2\ldots S_m)$$

where, n is the number of years of data and each transaction represents m different spare parts consumed together during maintenance. This dependency factor can also be judged on the confidence value of association rule among the spares. For example, confidence of rule $S_1 \rightarrow S_2$ should be greater than or equal to threshold or minimum confidence value which is decided based on expert judgments. Using similar procedure, one can compute dependency factor for three spare parts S_1, S_2 and S_3 and it can be computed as follows:

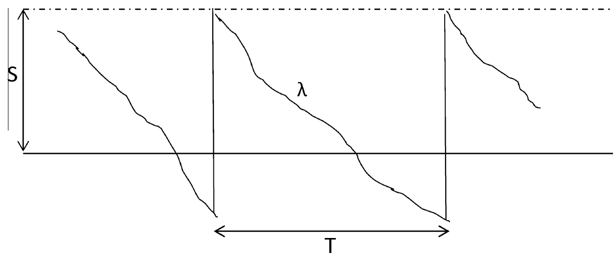

Fig. 1. Showing spare parts demand (λ), stock level (S) and cycle time (T).

Dependency factor among S_1 and S_2:

$$\alpha_{S_1S_2} = \frac{1}{n}\sum_{i=1}^{n} Support_i(S_1S_2) = \frac{1}{n}\sum_{i=1}^{n}\frac{|(S_1 \cup S_2)_i|}{|D_i|}$$

Similarly, Dependency factor among S_2 and S_3:

$$\alpha_{S_2S_3} = \frac{1}{n}\sum_{i=1}^{n} Support_i(S_2S_3) = \frac{1}{n}\sum_{i=1}^{n}\frac{|(S_2 \cup S_3)_i|}{|D_i|}$$

Dependency factor among S_1 and S_3:

$$\alpha_{S_1S_3} = \frac{1}{n}\sum_{i=1}^{n} Support_i(S_1S_3) = \frac{1}{n}\sum_{i=1}^{n}\frac{|(S_1 \cup S_3)_i|}{|D_i|}$$

Again, Dependency factor among S_1, S_2 and S_3:

$$\alpha_{S_1S_2S_3} = \frac{1}{n}\sum_{i=1}^{n} Support_i(S_1S_2S_3) = \frac{1}{n}\sum_{i=1}^{n}\frac{|(S_1 \cup S_2 \cup S_3)_i|}{|D_i|}$$

We have used the following example to demonstrate the procedure to compute the dependency factor. Suppose idler (S_1) and shaft (S_2) are the two spare parts that are consumed during the periodic maintenance of a belt conveyor. Table 2 shows transaction details for three years where the sign '$\surd$' indicates that the spare part is consumed and the sign '$\times$' indicates that it is not consumed. One can observe that both the spare parts are consumed together 4 times out of 7 transactions, 6 times out of 10 transactions and 6 times out of 9 transactions for first, second and third year respectively. Support value of spare parts (S_1, S_2) are 4/7, 6/10 and 6/9 for first, second and third year respectively. Dependency factor is calculated as the average of all support values i.e. 1/3[(4/7 + 6/10 + 6/9)] which is 0.6126. This value is different than the overall support value considering all three years together which is 16/26 i.e. 0.6154. In this study, we have used this dependency factor in the calculation of total cost of multi item inventory control to study the dependency effect.

3.2. (T, S) policy of inventory with shortages for a single spare part

Initially, we have considered here a single spare part for explaining the total cost function allowing shortages and using conventional periodic review (T, S) policy. In this system, at every T period, replenishment order is triggered to increase the order up to level (S). Fig. 1 shows the demand rate, order up-to level and cycle time of (T, S) policy with shortages. The total cost function presented here is with non-zero lead-time (refer Fung et al., 2001). Further, we have also assumed that spare parts demand arrival process is Poisson distributed. In addition, we have considered shortage penalty cost instead of service level constraints. In this model, the optimal order up-to level S will be determined using a periodic review (T, S) system with a given review period T where total cost (i.e., summation of ordering, holding and shortage) is minimum.

Following notation are used in the model development.

Notation:

i	1, 2, represents major and minor spare parts respectively
SPi	Spare part i
A_i	Ordering cost for the spare part i in \$.
H_i	Holding cost of spare part i in \$. per unit per year
λ_i	Annual demand of spare part i in units per year
λ'_j	Changed demand of spare part j due to dependency effect of spare part i
S_i	Order up-to level of spare part i in units
α_{ji}	Blockage of spare part j when spare part i is stock out, in units/unit
π_i	Maintenance loss cost due to unit shortage of spare part i
T_i	Cycle time of spare part i in years
L	Common lead time for both the spares
f_i	Percentage of demand filled from stock, fill rate of spare part i
μ	Mean of Poisson distribution
x	Random variable for Poisson distribution with p.d.f is $p(x)$ and c.d.f $P(x)$
TC_i	Total expected cost of spare part i (Rs./year)
TC	Total expected cost of all spares with common review period T(Rs./year)
OC	Ordering cost component (Rs./year)
HC	Holding cost component (Rs./year)
SC	Shortage cost component (Rs./year)
GTC	Total cost of all spare parts (Rs./year)

Referring Hadley and Whitin (1979), the total cost expression using Poisson distribution for single independent demand item can be written as:

$$Total\ annual\ cost = Ordering\ cost + holding\ cost + shortage\ cost$$

$$TC = \frac{A}{T} + h\left(\frac{\lambda T}{2} + \sum_{x_{T+L}=0}^{S}(S - x_{T+L})p(x_{T+L})\right) + \frac{SC}{T}\left(\sum_{x_{T+L}=S}^{\infty}(x_{T+L} - S)p(x_{T+L})\right)$$

where, lead time $L > 0$ and review period is T

The fill rate f can be defined as fraction of demand satisfied directly from the available stock. In periodic review system, the fill rate can be determined as follows:

$$f = 1 - \left(\frac{Expected\ shortage}{Available\ stock}\right) = 1 - \left(\frac{\sum_{x_T=S}^{\infty}(x_{T+L} - S)\cdot p(x_{T+L})}{\lambda(T+L)}\right)$$

where, $p(x_{T+L})$ is the probability mass function of the total demand x_{T+L} of spare part for the time period $T + L$. Considering Poisson distribution, $p(x_{T+L})$ can be written as:

$$p(x_{T+L}) = \frac{[\lambda(T+L)]^{x_{T+L}} \cdot e^{-[\lambda(T+L)]}}{x_{T+L}!}$$

4. Model development for two associated spare parts

We have developed the mathematical model for determining inventory control policy for two associated spare parts and called them as spare part 1(major item) and spare part 2 (minor item).

Further, it is assumed that cycle time of spare part-2 (minor item) will be elongated and made equal to spare part-1 when there is a shortage of spare part-1. In this model, we have considered (T, S) periodic review policy and tried to find out the order up to level (S) of each of the spare part with a given review period T. Based on different review period and fill rate (f_1) of spare part-1, maximum stock level (S_1) is determined first and keeping the same fill rate, the minimum total cost of second spare and maximum stock values (S_2) are estimated. If shortage is very less (i.e. fill rate is more), then amount of dependency will also be less for associated spares. The dependency effect of one spare on the other spares is shown in Fig. 2.The figure shows that when the demand of spare part-1 is satisfied with fraction f, rest of the demand will be on shortage with a fraction of $(1-f)$. Dependency effect starts from the end of Tf point and due to this effect, demand rate of spare part-2 will start declining up-to end of $T(1-f)$ period as shown in Fig. 2. The model is developed considering the following assumptions.

Assumptions:

In the development of the model, following assumptions are considered.

- Effect of shortages of dependent spare part is not negligible. This assumption is very much realistic in the sense that one cannot neglect the effect of associated minor spare parts on inventory stock due to the non availability of major spare parts.
- All shortage quantities of both the spares are lost maintenance cost. It is also a valid assumption since non availability of parts will ultimately have an impact on cost.
- Spares are associated to each other, and their dependencies are considered. This assumption is also valid in real scenario as it is observed that when one major item is consumed in the maintenance activity then along with it, certain minor items are also consumed and hence always there exists a dependency effect.
- Unit shortage of spare part-1 (major) will reduce the demand of spare part-2 (minor) at a constant rate. It is appropriate to consider this assumption because spare parts are consumed in the group in periodic maintenance activities and there exists certain dependency among these spare parts. The rate of dependency can be linear and non-linear. In this model, we have limited our study with linear (constant) dependency using support value for illustration of our model.

Table 3
Optimal order up-to level and total cost with T = 0.4 year with L = 0 and L = 1/12 year.

Spare part-1 ($f > 0.95$)				Spare part-2 ($f > 0.95$)			
L = 0		L = 1/12 year		L = 0		L = 1/12 year	
S_1	TC_1	S_1	TC_1	S_2	TC_2	S_2	TC_2
80	2566.86	96	2661.74	116	1792.14	144	1777.76
81	2518.66	97	2608.46	117	1780.44	145	1769.64
82	2476.85	98	2561.04	118	1769.97	146	1762.68
83	2441.26	99	2519.42	119	1760.78	147	1756.87
84	2411.67	100	2483.47	120	1752.85	148	1752.21
85	2387.80	101	2453.00	121	1746.20	149	1748.65
86	2369.30	102	2427.79	122	1740.81	150	1746.19
87	2355.81	103	2407.57	123	1736.67	151	1744.78
88	2346.92	104	2392.03	124	1733.73	**152**	**1744.38**
89	2342.21	105	2380.86	125	1731.97	153	1744.95
90	**2341.26**	106	2373.72	**126**	**1731.34**	154	1746.44
91	2343.65	107	2370.24	127	1731.78	155	1748.80
92	2348.98	**108**	**2370.10**	128	1733.23	156	1751.96
93	2356.86	109	2372.93	129	1735.63	157	1755.88
94	2366.94	110	2378.40	130	1738.92	158	1760.50
95	2378.90	111	2386.21	131	1743.02	159	1765.76
96	2392.42	112	2396.04	132	1747.87	160	1771.61
97	2407.26	113	2407.64	133	1753.40	161	1777.99
98	2423.18	114	2420.73	134	1759.54	162	1784.84
99	2439.99	115	2435.10	135	1766.23	163	1792.13

4.1. Mathematical model

Referring Hadley and Whitin (1979), the total annual costs of major spare parts comprise ordering costs, inventory carrying costs and cost due to shortages. Using Poisson distributed demand, total expected cost expression of spare part 1 can be written as:

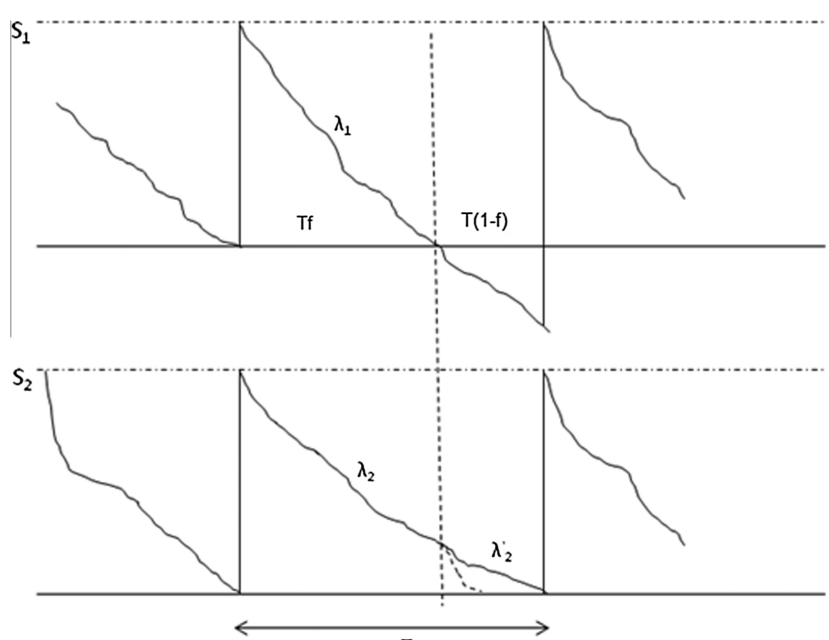

Fig. 2. Stock levels of dependent spare parts with common review period.

$$TC(SP1) = \left(\frac{A_1}{T_1}\right) + H_1\left[\frac{\lambda_1 T_1}{2} + \sum_{x_1=0}^{S_1}\left[(S_1 - x_1)\cdot\frac{[\lambda_1(T_1+L)]^{x_1}\cdot e^{-\lambda_1(T_1+L)_1}}{x_1!}\right]\right] + \frac{\pi_1}{T_1}\sum_{x_1=S_1+1}^{\infty}\left[(x_1 - S_1)\cdot\frac{[\lambda_1(T_1+L)]^{x_1}\cdot e^{-\lambda_1(T_1+L)}}{x_1!}\right] \quad (1)$$

Fill rate f_1 can be written as:

$$f_1 = 1 - \left(\frac{\sum_{x_1=S_1+1}^{\infty}\left[(x_1 - S_1)\cdot\frac{[\lambda_1(T_1+L)]^{x_1}\cdot e^{-\lambda_1(T_1+L)}}{x_1!}\right]}{\lambda_1(T_1+L)}\right) \quad (2)$$

Now for the spare part 2, $\lambda_2' = \lambda_2 - \alpha_{21}\lambda_1$ and α_{21} is the dependency factor which can be calculated by following the procedure described in the Section 3.1. By using the value of λ_2', the different components of total expected cost of dependent spare part (spare part 2) can be calculated as follows:

Ordering cost = $\left(\frac{A_2}{T_2}\right)$
Holding cost = $\frac{H_2}{T_2}\left[\frac{1}{2}\lambda_2 f_1 T_1\cdot f_1 T_1 + \frac{1}{2}\lambda_2'(T_2 - f_1T_1)\cdot f_1T_1 + \frac{1}{2}\lambda_2'(T_2 - f_1T_1)\cdot\lambda_2'(T_2 - f_1T_1)\right] + H_2[\textit{Probablistic part of inventory holding}]$
Shortage cost = $\frac{\pi_2}{T_2}[\textit{Shortages}]$

Total expected cost of dependent spare part (spare part 2) can be written as:

$$TC(SP2) = \left(\frac{A_2}{T_2}\right) + H_2\left[\frac{\lambda_2 f_1^2 T_1^2}{2T_2} + \frac{\lambda_2'(T_2 - f_1T_1)f_1T_1}{2T_2} + \frac{\lambda_2'(T_2 - f_1T_1)^2}{2T_2}\right] + H_2\left[\sum_{x_2=0}^{\lambda_2'((T_2+L)-f_1(T_1+L))}(S_2 - x_2)\cdot\frac{[\lambda_2'((T_2+L)-f_1(T_1+L))]^{x_2}\cdot e^{-[\lambda_2'((T_2+L)-f_1(T_1+L))]}}{x_2!}\right] + H_2\left[\sum_{x_2=\lambda_2'((T_2+L)-f_1(T_1+L))+1}^{S_2}(S_2 - x_2)\cdot\frac{[\lambda_2 f_1(T_1+L)]^{x_2}\cdot e^{-[\lambda_2 f_1(T_1+L)]}}{x_2!}\right] + \frac{\pi_2}{T_2}\left[\sum_{x_2=S_2+1}^{\infty}(x_2 - S_2)\cdot\frac{[\lambda_2'(T_2+L)]^{x_2}\cdot e^{-[\lambda_2'(T_2+L)]}}{x_2!}\right] \quad (3)$$

Now, the total expected cost comprising of these two spares can be written as:

$$TC = TC(SP1) + TC(SP2)$$

In practice, the major spare part is considered as critical part with high unit price and less demand rate as compared to minor parts. Hence, the assumption is made that the review period of major spare part is higher than the one for minor spare parts. In this model, when major spare part faces shortages, the review period of minor spare part is likely to be more elongated up-to the review period of major spare parts. At this point, keeping both the spare parts with common review period T i.e. $T = T_1 = T_2$, total expected cost can be written as:

$$TC = \left(\frac{A_1 + A_2}{T}\right) + H_1\left[\frac{\lambda_1 T}{2} + \sum_{x_1=0}^{S_1}(S_1 - x_1)\cdot\sum_{x_1=0}^{S_1}\left[(S_1 - x_1)\cdot\frac{[\lambda_1(T+L)]^{x_1}\cdot e^{-\lambda_1(T+L)}}{x_1!}\right]\right] + \frac{H_2 T}{2}\left[\lambda_2 f_1^2 + \lambda_2'(1-f_1)f_1 + \lambda_2'(1-f_1)^2\right] + H_2\left[\sum_{x_2=0}^{\lambda_2'(T+L)(1-f_1)}\left[(S_2 - x_2)\cdot\frac{[\lambda_2'(T+L)(1-f_1)]^{x_2}\cdot e^{-[\lambda_2'(T+L)(1-f_1)]}}{x_2!}\right]\right] + H_2\left[\sum_{x_2=\lambda_2'(T+L)(1-f_1)+1}^{S_2}\left[(S_2 - x_2)\cdot\frac{[\lambda_2 f_1(T+L)]^{x_2}\cdot e^{-[\lambda_2 f_1(T+L)]}}{x_2!}\right]\right] + \frac{\pi_1}{T}\sum_{x_1=S_1+1}^{\infty}\left[(x_1 - S_1)\cdot\frac{[\lambda_1(T+L)]^{x_1}\cdot e^{-\lambda_1(T+L)}}{x_1!}\right] + \frac{\pi_2}{T}\left[\sum_{x_2=S_2+1}^{\infty}(x_2 - S_2)\cdot\frac{[\lambda_2'(T+L)]^{x_2}\cdot e^{-[\lambda_2'(T+L)]}}{x_2!}\right] \quad (4)$$

The expected shortage cost expression can be further simplified and presented in the following form:

$$\sum_{x=S+1}^{\infty}(x-S)\cdot\frac{[\lambda T]^x\cdot e^{-[\lambda T]}}{x!} = \sum_{x=0}^{\infty}(x-S)\cdot\frac{[\lambda T]^x\cdot e^{-[\lambda T]}}{x!} - \sum_{x=0}^{S}(x-S)\cdot\frac{[\lambda T]^x\cdot e^{-[\lambda T]}}{x!} = \sum_{x=0}^{\infty}x\cdot\frac{[\lambda T]^x\cdot e^{-[\lambda T]}}{x!} - S\cdot\sum_{x=0}^{\infty}\frac{[\lambda T]^x\cdot e^{-[\lambda T]}}{x!} - \sum_{x=0}^{S}(x-S)\cdot\frac{[\lambda T]^x\cdot e^{-[\lambda T]}}{x!} = \mu - S + \sum_{x=0}^{S}(S-x)\cdot\frac{[\lambda T]^x\cdot e^{-[\lambda T]}}{x!} = \lambda T - S + \sum_{x=0}^{S}(S-x)\cdot\frac{[\lambda T]^x\cdot e^{-[\lambda T]}}{x!} \quad (5)$$

5. Solution algorithm

The above problem is a non-linear mixed integer programming problem which is difficult to solve mathematically to get the stock level (S) values of the spares. Hence, we proposed a heuristic method such as exhaustive search to solve the problem for getting near optimal solution. This method is a simple and widely adopted problem solving technique which enumerates possible candidates systematically and then check for the solution. In each of the solution, it checks whether each candidate satisfies the problem statement. This method is called as generate and test method, where each scenario is generated and tested simultaneously to get the optimum solution. It is also considered as standard algorithm for computational tasks such as solving non linear equations, and finding a maximum or minimum in a list with or without constraints.

In this algorithm (5.1), only an integer value of S is enumerated from 1 to a maximum value set by the user. It takes $A_1, A_2, h_1, h_2, \pi_1, \alpha_{21}, \lambda_1, f_1, \lambda_2, T$ and L as input values for the parameters to compute the total costs of both the spare parts. This algorithm uses minimum total cost value as termination criteria to come out from the loop. In the algorithm 5.1, Step 2 explains about the enumeration of S value from 1 to a maximum user defined value. For each S value, the total cost function of a major spare part and its fill rate are computed. If the total cost is minimum and the calculated fill rate is greater than or equal to the user specified value then the loop exits. In Step 5, the total cost of a minor spare part is computed using the calculated fill rate of the major spare part with a similar enumeration method as Step 3. The grand total cost (TC) is computed using the minimum total cost of major and minor cost.

The second algorithm (5.2) searches for optimal total costs and order up-to level values by enumerating the review period and fill rate values. The initial values of the review period and fill rate are taken as 0.05 years and 0.05 respectively and these values are incremented by an increment of 0.05 for each iteration. The review period enumerates from 0.05 to a maximum value of 2 years and fill rates from 0.05 to 0.99. The outcome of the algorithm gives

Table 4
Optimal stock level and cost for different range of review periods.

T	Spare part-1 ($f > 0.95$)				Spare part-2 ($f > 0.95$)			
	$L = 0$		$L = 1/12$ year		$L = 0$		$L = 1/12$ year	
	S_1^*	TC_1^*	S_1^*	TC_1^*	S_2^*	TC_2^*	S_2^*	TC_2^*
0.3	70	2540.27	88	2575.29	97	1909.88	123	1926.32
<u>0.4</u>	<u>90</u>	<u>2341.26</u>	<u>108</u>	<u>2370.10</u>	<u>126</u>	<u>1731.34</u>	<u>152</u>	<u>1744.38</u>
0.5	**110**	**2304.51**	**127**	**2328.64**	**155**	**1684.30**	**181**	**1695.11**
0.6	129	2348.06	147	2368.90	184	1702.80	210	1712.04
0.7	149	2436.82	166	2455.65	213	1758.67	238	1766.72
0.8	169	2554.05	186	2570.69	242	1837.86	267	1844.94
0.9	188	2689.90	206	2705.15	271	1932.58	296	1938.88
1.0	208	2838.58	225	2852.42	300	2038.16	325	2043.84
1.1	227	2996.81	245	3009.62	329	2151.64	354	2156.81
1.2	247	3161.76	264	3173.58	358	2271.05	383	2275.78

the optimal review period, order up-to level values, and fill rate. Both the algorithms (5.1 and 5.2) are explained in steps below.

5.1. Algorithm for computing order up to level (S) and total cost (TC) values

Step 0	Initialize Given $A_1, A_2, h_1, h_2, \pi_1, \alpha_{21}, \lambda_1,\ f_1, \lambda_2, T, L$ $S_1^{old} = 1, S_2^{old} = 1, S_1^{new} = 1, S_2^{new} = 1, f = 0$
Step 1	Set maximum value of $S_1^{\max} = \infty$
Step 2	While $S_1^{old} \leqslant S_1^{\max}$ Compute $TC(SP1)^{new}$ *and* f using Eqs. (1) and (2) respectively $TC(SP1)^{old} = TC(SP1)^{new}$ $S_1^{old} = S_1^{old} + 1$
Step 3	Stopping the loop If $TC(SP1)^{new} \leqslant TC(SP1)^{old}$ *and* $f \geqslant f_1$ Then $S_1^{old} = S_1^{new}$ $TC(SP1)^{old} = TC(SP1)^{new}$ Else return to the Step 2.
Step 4	Return the value of $S_1^* = S_1^{old}$ and $TC(SP1)^* = TC(SP1)^{old}$
Step 5	Use this f_1 value in total cost Eq. (3) and use the similar steps from 1 to 4 to optimal value of S_2^{old} and $TC(SP2)^* = TC(SP2)^{old}$
Step 6	Return $S_1^* = S_1^{old},\ S_2^* = S_2^{old}$ and $TC^* = TC(SP1)^* + TC(SP2)^*$ values for common T

The same algorithm can also be used for computing total cost for various range of *T*, *f*.

5.2. Algorithm for computing S and TC values for different ranges of T and f

Step 0	Initialize $T^{\min} = 0.05,\ T^{\max} = 2.00,\ f^{\min} = 0.05,\ f^{\max} = 0.99$
Step 1	While $T^{\min} \leqslant T^{\max}$ While $f^{\min} \leqslant f^{\max}$ Compute the S^* and TC^* using first algorithm with $T^{\min}$ and $f^{\min}$ values $T^{\min} = T^{\min} + 0.05$ $f^{\min} = f^{\min} + 0.05$
Step 2	Return all values of S^* and TC^*

Complexity:

The developed model is non linear in nature. Hence, we use heuristic procedure to solve the problem. In this model decision variables are always integers and a near optimal solution is achieved using the algorithm (5.1). The proposed algorithm is fail safe and handle exceptions in case of not a number (NaN) values, division by zero and negative number inside the loops. The algorithms are implemented using Java programming language with Intel(R) Core (TM) i5-3320 M CPU, 2.60 GHz processor, 8 GB RAM, 64-bit operating system. In this case the algorithm is comprising a loop statement iterating over a range R. Hence, the complexity of the algorithm is O(R) where O(R) will be worst possible case and O(1) will be best possible case.

Termination:

A termination criterion plays an important role for the correctness and performance of an algorithm. If the size of data input is fixed, the loop is definitely liable to terminate. Our model uses fixed data input for finding order up to levels and total cost values. In this algorithm, we have used minimum total cost and fill rate as termination criteria in the case of major spare parts. Secondly, only the minimum total cost is used as the termination criteria for minor spare parts.

Limitations of the algorithm:

As mentioned earlier, the developed model is non linear in nature and practically difficult to find the optimal values of stock levels and total cost. Hence, we proposed exhaustive search algorithm to solve this problem. This algorithm enumerates specific input parameters and search for local optimal values comparing with previous values. This search algorithm is feasible only if the search tree grows in a binary fashion.

The proposed algorithm uses integer decision variables because the model uses Poisson distribution function which is discrete in nature. This approach also generates large number of candidate solutions when problem size is more and cost of finding optimal solution will be higher due to computational complexity.

6. Numerical example

We have considered the following numerical example to illustrate the application of the suggested model for two associated spare parts. The approximate solutions for the proposed models are computed using the algorithm given in this study. We have computed the total cost values for lead-time = 0 and 1/12 years.

Spare part-1(SP1)	Spare part-2(SP2)
A_1 = Rs. 500/order	A_2 = Rs. 400/order
H_1 = Rs. 20/unit/year	H_2 = Rs. 10/unit/year
λ_1 = 200 units/year	λ_2 = 300 units/year
π_1 = Rs. 50/unit/year	π_2 = Rs. 10/unit/year
$\alpha_{21} = 0.3$	

Case-1: Periodic review inventory control for single spare part:

In this case, we have tried to find out the maximum order up to level (*S*) of spare part allowing shortages. Various *S* values are computed with respect to *T* values using enumeration method as shown in Table 3. In this method, *S* values are incremented by one unit and correspondingly check the point where the total cost is minimum. One can observe that additional quantity of 18 and 26 of spare part-1 and spare part-2 are to be stocked respectively if the lead-time 1/12 year is introduced.

We have used the review period of 0.4 years to compute the stock level of each spare, which is under lined in Table 4. The results of optimal stock levels and total cost values for a different range of review periods *T* starting from 0.3 years to 1.2 years are provided in Table 4. From the results, it is observed that lowest optimal stock level 110 and cost Rs. 2304.51 falls in the review period of 0.5 years for spare part-1. Considering the lead-time of 1/12 year, stock level of 127 can be maintained with the total inventory related cost of Rs. 2328.64 for same review period of 0.5 years. Similarly, for the second spare part, optimal stock level of 155 and 181 can be maintained with total cost of Rs. 1684.30 and Rs. 1695.11 taking zero lead-time and lead-time of 1/12 year respectively. These order up-to level values for both the spare parts are calculated based on the service level constraint, i.e. fill rate of 0.95. With lesser threshold values of f, order up to level values are less with high total cost values due to increase of the shortage cost.

Table 5
Joint optimal total cost values of spare for different review periods and fill rates with dependency factor $\alpha = 0.3$.

	Fill rate (f)														
T	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95	0.99
0.05	25385.78	25381.37	24387.54	24383.20	23408.72	23404.45	22468.65	22464.50	21593.23	21589.39	20810.42	20146.63	19610.85	18913.87	18536.33
0.10	16648.00	16139.56	15631.51	15124.22	14618.55	14116.01	13619.16	13131.80	12659.09	12207.40	11783.76	11394.76	10744.69	10278.81	9798.23
0.15	13902.86	13557.05	12878.16	12532.82	11854.74	11510.42	10836.36	10496.90	9840.91	9519.21	8915.62	8375.07	7920.76	7427.46	7067.71
0.20	12652.08	12135.97	11620.21	11104.77	10589.69	10075.19	9561.96	9051.87	8548.99	8060.41	7595.44	7167.44	6619.21	6111.40	5809.47
0.25	11996.97	11577.44	10958.44	10539.92	9921.89	9504.42	8888.04	8473.78	7866.64	7467.37	6897.39	6376.78	5933.46	5498.66	5174.35
0.30	11638.91	11116.20	10594.25	10072.99	9552.41	9032.58	8513.91	7997.94	7488.01	6989.78	6512.36	6069.87	5559.36	5105.79	4831.23
0.35	11449.20	10994.96	10398.90	9946.64	9352.44	8902.12	8310.51	7864.98	7283.58	6855.78	6307.71	5927.14	5366.35	4965.24	4676.07
0.40	**11363.36**	10834.97	10307.95	9782.15	9257.55	8734.28	8213.14	7696.54	7188.34	**6692.82**	6217.13	**5774.31**	**5289.19**	**4871.91**	**4633.31**
0.45	11455.62	10868.99	**10284.26**	9812.22	**9230.61**	8761.92	**8185.35**	7726.24	**7166.02**	6730.46	**6204.22**	5815.76	5291.33	4916.50	4663.42
0.50	11467.88	**10832.62**	10300.60	**9771.25**	9244.78	**8721.79**	8204.02	**7694.14**	7195.40	6711.24	6246.63	5812.60	5430.25	4963.24	4754.15
0.55	11414.10	10911.75	10321.03	9830.57	9269.21	8805.20	8253.34	7798.30	7262.56	6832.13	6330.64	5946.12	5526.99	5102.88	4887.27
0.60	11560.68	10915.58	10357.02	9816.07	9309.91	8815.76	8324.59	7837.50	7358.46	6893.05	6446.60	6028.24	5658.43	5229.97	5053.80
0.65	11633.10	11019.23	10409.65	9903.15	9367.74	8927.44	8413.78	7980.66	7476.90	7062.00	6588.30	6216.34	5818.09	5427.69	5247.57
0.70	11719.75	11059.62	10476.58	9928.25	9440.33	8977.11	8518.24	8063.15	7613.38	7173.77	6751.07	6353.59	6000.92	5646.56	5464.15
0.75	11818.12	11183.24	10555.55	10037.92	9525.44	9111.23	8635.71	8231.06	7764.71	7373.69	6931.48	6578.21	6203.42	5845.40	5700.23
0.80	11926.39	11249.73	10644.73	10091.64	9621.25	9189.53	8764.35	8343.95	7928.60	7520.91	7126.91	6754.71	6422.76	6098.65	5953.33
0.85	12042.74	11386.92	10742.63	10217.45	9726.28	9339.84	8902.63	8529.57	8103.21	7742.66	7335.36	7005.72	6656.84	6331.37	6221.44
0.90	12165.80	11471.90	10847.95	10292.50	9839.33	9439.22	9049.12	8665.04	8287.04	7915.84	7555.11	7211.75	6903.80	6610.97	6504.95
0.95	12294.12	11617.88	10959.84	10430.27	9959.40	9600.86	9202.47	8863.47	8478.59	8153.34	7784.46	7482.38	7161.83	6869.93	6798.38
1.00	12425.01	11714.73	11077.52	10521.68	10085.70	9714.28	9360.01	9013.75	8675.00	8343.77	8020.46	7708.75	7427.67	7167.83	7100.31
1.05	12545.50	11863.28	11200.22	10668.85	10217.56	9879.06	9506.72	9190.24	8834.37	8534.35	8195.21	7914.97	7617.78	7353.63	7313.08
1.10	12622.17	11966.46	11327.53	10773.47	10354.47	9999.42	9656.75	9323.10	8998.44	8682.76	8376.30	8081.76	7816.43	7577.19	7536.27
1.15	12792.57	12118.04	11459.00	10928.28	10495.99	10169.44	9811.42	9506.71	9168.31	8883.21	8564.56	8301.41	8024.06	7783.34	7769.92
1.20	12920.41	12229.03	11594.16	11043.86	10641.77	10300.24	9970.82	9652.09	9344.02	9046.58	8759.89	8486.06	8240.40	8025.15	8013.64
1.25	13051.02	12384.55	11732.80	11205.12	10791.55	10476.89	10134.79	9844.31	9525.36	9257.98	8962.21	8719.56	8465.36	8275.65	8266.99
1.30	13184.17	12502.73	11874.66	11330.05	10945.05	10617.54	10303.24	10001.48	9712.25	9435.48	9171.17	8920.93	8698.51	8510.18	8529.85
1.35	13319.79	12661.97	12019.42	11496.89	11102.09	10800.61	10476.01	10202.01	9904.49	9657.40	9386.73	9167.70	8939.74	8777.60	8801.99
1.40	13457.62	12786.52	12167.01	11629.91	11262.51	10950.30	10652.96	10370.18	10101.89	9848.05	9608.68	9384.86	9188.87	9030.17	9083.19
1.45	13597.59	12949.09	12317.20	11801.76	11426.17	11139.39	10833.95	10578.61	10304.37	10080.14	9836.87	9644.40	9445.70	9313.94	9373.30
1.50	13739.56	13079.31	12469.85	11942.04	11592.95	11297.47	11018.86	10757.00	10511.78	10283.16	10071.16	9876.43	9710.05	9583.86	9672.15
1.55	13883.41	13244.94	12624.89	12118.48	11762.66	11492.27	11207.61	10973.05	10724.03	10525.01	10311.42	10148.34	9981.76	9883.29	9979.60
1.60	14029.04	13380.22	12782.13	12265.33	11935.30	11658.12	11400.09	11161.03	10941.00	10739.82	10557.55	10394.62	10260.77	10169.98	10295.56
1.65	14176.35	13548.67	12941.56	12446.03	12110.76	11858.47	11596.19	11384.45	11162.60	10991.18	10809.48	10678.54	10546.95	10484.71	10619.90
1.70	14325.25	13688.57	13103.03	12598.89	12288.90	12031.63	11795.84	11581.58	11388.74	11217.26	11067.12	10938.59	10840.22	10787.53	10952.54
1.75	14475.67	13859.60	13266.52	12783.53	12469.75	12237.25	11999.01	11812.11	11619.37	11477.88	11330.40	11234.26	11140.49	11117.36	11293.41
1.80	14627.55	14003.74	13431.94	12941.94	12653.15	12417.30	12205.63	12018.00	11854.43	11714.88	11599.23	11507.65	11447.69	11435.92	11642.42
1.85	14780.82	14177.27	13599.23	13130.42	12839.13	12628.10	12415.57	12255.56	12093.87	11984.58	11873.55	11814.91	11761.73	11780.58	11999.52
1.90	14935.43	14325.27	13768.35	13294.08	13027.58	12814.76	12628.86	12469.84	12337.63	12232.17	12153.33	12101.29	12082.57	12114.56	12364.60
2.00	15091.34	14501.12	13939.24	13486.24	13218.49	13030.60	12845.47	12714.22	12585.64	12510.84	12438.51	12419.99	12410.15	12473.75	12731.89

Table 6
Showing order up-to level values for both spares with $T = 0.4$, $\alpha = 0.0$ and $\alpha = 0.3$ ($L = 0$).

Dependency factor $\alpha = 0.0$ (No dependency)								Dependency factor $\alpha = 0.3$							Cost savings (%)
f			*OC*	*HC*	*SC*	*TC*	*GTC*			*OC*	*HC*	*SC*	*TC*	*GTC*	
0.30	S_1	24	1250.00	800.00	7000.00	9050.00	11787.33	S_1	24	1250.00	800.00	7000.00	9050.00	11363.36	3.60
	S_2	118	1000.00	1601.63	135.70	2737.33		S_2	88	1000.00	1085.82	227.55	2313.36		
0.35	S_1	28	1250.00	800.00	6500.00	8550.00	11287.33	S_1	28	1250.00	800.00	6500.00	8550.00	10834.97	4.01
	S_2	118	1000.00	1601.63	135.70	2737.33		S_2	88	1000.00	1057.43	227.55	2284.97		
0.40	S_1	32	1250.00	800.00	6000.00	8050.00	10787.33	S_1	32	1250.00	800.00	6000.00	8050.00	10307.95	4.44
	S_2	118	1000.00	1601.63	135.70	2737.33		S_2	88	1000.00	1030.40	227.55	2257.95		
0.45	S_1	36	1250.00	800.00	5500.00	7550.00	10287.33	S_1	36	1250.00	800.00	5500.00	7550.00	9782.15	4.91
	S_2	118	1000.00	1814.15	135.70	2737.33		S_2	88	1000.00	1004.60	227.55	2232.15		
0.50	S_1	40	1250.00	800.00	5000.00	7050.00	9787.33	S_1	40	1250.00	800.00	5000.00	7050.00	9257.55	5.41
	S_2	118	1000.00	1601.63	135.70	2737.33		S_2	88	1000.00	980.01	227.55	2207.55		
0.55	S_1	44	1250.00	800.00	4500.00	6550.00	9287.33	S_1	44	1250.00	800.00	4500.00	6550.00	8734.28	5.95
	S_2	118	1000.00	1601.63	135.70	2737.33		S_2	88	1000.00	956.73	227.55	2184.28		
0.60	S_1	48	1250.00	800.00	4000.01	6050.01	8787.34	S_1	48	1250.00	800.00	4000.01	6050.01	8213.14	6.53
	S_2	118	1000.00	1601.63	135.70	2737.33		S_2	88	1000.00	935.58	227.55	2163.13		
0.65	S_1	52	1250.00	800.02	3500.11	5550.13	8287.46	S_1	52	1250.00	800.02	3500.11	5550.13	7696.54	7.13
	S_2	118	1000.00	1601.63	135.70	2737.33		S_2	89	1000.00	938.27	208.15	2146.42		
0.70	S_1	56	1250.00	800.11	3000.70	5050.81	7788.14	S_1	56	1250.00	800.11	3000.70	5050.81	7188.34	7.70
	S_2	118	1000.00	1601.63	135.70	2737.33		S_2	91	1000.00	965.69	171.85	2137.53		
0.75	S_1	60	1250.00	800.55	2503.44	4553.99	7291.32	S_1	60	1250.00	800.55	2503.44	4553.99	6692.82	8.21
	S_2	118	1000.00	1601.63	135.70	2737.33		S_2	92	1000.00	983.79	155.04	2138.83		
0.80	S_1	64	1250.00	802.13	2013.33	4065.46	6802.79	S_1	64	1250.00	802.13	2013.33	4065.46	6217.13	8.61
	S_2	118	1000.00	1601.63	135.70	2737.33		S_2	94	1000.00	1027.33	124.33	2151.66		
0.85	S_1	68	1250.00	806.68	1541.76	3598.44	6335.77	S_1	68	1250.00	806.68	1541.76	3598.44	5774.31	8.86
	S_2	118	1000.00	1601.63	135.70	2737.33		S_2	96	1000.00	1078.24	97.64	2175.87		
0.90	S_1	73	1250.00	821.33	1008.33	3079.66	5816.99	S_1	73	1250.00	821.33	1008.33	3079.66	5289.19	9.07
	S_2	118	1000.00	1601.63	135.70	2737.33		S_2	97	1000.00	1123.71	85.81	2209.52		
0.95	S_1	79	1250.00	861.59	509.93	2621.51	5358.84	S_1	79	1250.00	861.59	509.93	2621.51	4871.91	**9.09**
	S_2	118	1000.00	1601.63	135.70	2737.33		S_2	98	1000.00	1175.40	75.00	2250.40		
0.99	S_1	88	1250.00	978.88	118.03	2346.92	5084.25	S_1	88	1250.00	978.88	118.03	2346.92	4633.31	8.87
	S_2	118	1000.00	1601.63	135.70	2737.33		S_2	98	1000.00	1211.39	75.00	2286.39		

Case-2: Periodic review model for associated spares:

In this case, we used the (T, S) periodic review model and tried to estimate the effect of shortages of first spare on the second spare when they are associated. T is the review period, and S is the order up to level or maximum stock level. Initially, we computed the fill rate (f_1) and maximum stock level (S_1) for first spare or major spare allowing shortages with given review period. Then we use this f_1 value to estimate the minimum total cost of second spare and maximum stock values (S_2). If there is no shortage of first spare, then the amount of dependency of first to second spares are ignored. Dependency effect begins from the start point of shortages of first spare only i.e. demand of second spare starts declining with constant rate at the shortage point or zero stock point of major spare. We have also studied the effect stock levels of associated spares if there is non-zero lead-time (orders are not instantaneous).

We have used the same data set which was used for the previous example and computed joint optimal total cost for both the spares as shown in Table 5. This table shows optimal total cost of these two spare parts with respect to fill rate (0.30–0.99) and review periods (0.05–2.00). We observed that most of the values are found in the range of review periods between 0.4 and 0.5 years with dependency factor 0.3 (as shown in bold letters). Next, we have determined order up-to level values for non-associated $(\alpha = 0)$ and associated spares $(\alpha = 0.3)$ with lead time zero and lead-time of 1/12 years. These details are given in Tables 6 and 7 for different ranges of fill rate ($f_1 = 0.30$ to 0.99). We observed that for non-associated spares, stock level of second spare is constant for different ranges of f_1 values as there is no relation between them (118 when $L = 0$ and 143 when $L = 1/12$). But in case of first spare, stock value increases with fill rate (24 to 88 when $L = 0$ and 29 to 105 when $L = 1/12$) and total cost decreases with fill rate (9050.00 to 2346.92 when $L = 0$ and 10525.00 to 2380.86 when $L = 1/12$) as we are stocking more quantities to reduce the shortages which in turn reduces the total cost. This is because the shortage costs of spare parts are higher as compared to holding cost.

But when we introduce a dependency factor $\alpha = 0.3$, stock level of second spare starts to increase from 88 to 98 with the range of f_1 from 0.30 to 0.99 for zero lead time. Comparing with non-associated spare parts, we have observed that percentage savings of total cost of inventory starts increasing (3.60–9.09%) from fill rate 0.30 to 0.95. The cost saving decreases at fill rate 0.99 (8.87%), because holding cost of dependent spare part (SP2) increases from Rs. 1175.40 to Rs. 1211.39. Similarly, we get the

Table 7
Showing order up-to level values for both spares with $T = 0.4$, $\alpha = 0.0$ and $\alpha = 0.3$ ($L = 1/12$ years).

Dependency factor $\alpha = 0.0$ (No dependency)								Dependency factor $\alpha = 0.3$							Cost savings (%)
f			*OC*	*HC*	*SC*	*TC*	*GTC*			*OC*	*HC*	*SC*	*TC*	*GTC*	
0.30	S_1	29	1250.00	800.00	8475.00	10525.00	13523.95	S_1	29	1250.00	800.00	8475.00	10525.00	13031.56	3.64
	S_2	143	1000.00	1849.64	149.31	2998.95		S_2	107	1000.00	1248.97	257.58	2506.56		
0.35	S_1	33	1250.00	800.00	7975.00	10025.00	13023.95	S_1	33	1250.00	800.00	7975.00	10025.00	12500.75	4.02
	S_2	143	1000.00	1849.64	149.31	2998.95		S_2	107	1000.00	1218.17	257.58	2475.75		
0.40	S_1	38	1250.00	800.00	7350.00	9400.00	12398.95	S_1	38	1250.00	800.00	7350.00	9400.00	11846.22	4.46
	S_2	143	1000.00	1849.64	149.31	2998.95		S_2	107	1000.00	1188.64	257.58	2446.22		
0.45	S_1	43	1250.00	800.00	6725.00	8775.00	11773.95	S_1	43	1250.00	800.00	6725.00	8775.00	11192.90	4.94
	S_2	143	1000.00	1849.64	149.31	2998.95		S_2	107	1000.00	1160.32	257.58	2417.90		
0.50	S_1	48	1250.00	800.00	6100.00	8150.00	11148.95	S_1	48	1250.00	800.00	6100.00	8150.00	10540.78	5.45
	S_2	143	1000.00	1849.64	149.31	2998.95		S_2	107	1000.00	1133.20	257.58	2390.78		
0.55	S_1	53	1250.00	800.00	5475.00	7525.00	10523.95	S_1	53	1250.00	800.00	5475.00	7525.00	9889.91	6.02
	S_2	143	1000.00	1849.64	149.31	2998.95		S_2	107	1000.00	1107.33	257.58	2364.91		
0.60	S_1	58	1250.00	800.00	4850.00	6900.00	9898.95	S_1	58	1250.00	800.00	4850.00	6900.00	9240.85	6.64
	S_2	143	1000.00	1849.64	149.31	2998.95		S_2	107	1000.00	1083.26	257.58	2340.84		
0.65	S_1	62	1250.00	800.00	4350.02	6400.02	9398.97	S_1	62	1250.00	800.00	4350.02	6400.02	8720.83	7.21
	S_2	143	1000.00	1849.64	149.31	2998.95		S_2	108	1000.00	1082.92	237.89	2320.81		
0.70	S_1	67	1250.00	800.03	3725.21	5775.24	8774.19	S_1	67	1250.00	800.03	3725.21	5775.24	8084.53	7.86
	S_2	143	1000.00	1849.64	149.31	2998.95		S_2	110	1000.00	1108.59	200.70	2309.29		
0.75	S_1	72	1250.00	800.24	3101.49	5151.73	8150.68	S_1	72	1250.00	800.24	3101.49	5151.73	7462.17	8.45
	S_2	143	1000.00	1849.64	149.31	2998.95		S_2	112	1000.00	1143.71	166.73	2310.44		
0.80	S_1	77	1250.00	801.26	2482.88	4534.14	7533.09	S_1	77	1250.00	801.26	2482.88	4534.14	6860.56	8.93
	S_2	143	1000.00	1849.64	149.31	2998.95		S_2	114	1000.00	1190.18	136.24	2326.42		
0.85	S_1	82	1250.00	805.04	1881.48	3936.52	6935.47	S_1	82	1250.00	805.04	1881.48	3936.52	6293.45	9.26
	S_2	143	1000.00	1849.64	149.31	2998.95		S_2	116	1000.00	1247.53	109.40	2356.93		
0.90	S_1	88	1250.00	819.11	1219.42	3288.53	6287.48	S_1	88	1250.00	819.11	1219.42	3288.53	5688.19	9.53
	S_2	143	1000.00	1849.64	149.31	2998.95		S_2	117	1000.00	1302.29	97.37	2399.66		
0.95	S_1	95	1250.00	861.50	609.40	2720.91	5719.86	S_1	95	1250.00	861.50	609.40	2720.91	5171.57	**9.58**
	S_2	143	1000.00	1849.64	149.31	2998.95		S_2	118	1000.00	1364.41	86.26	2450.66		
0.99	S_1	105	1250.00	987.02	143.85	2380.86	5379.81	S_1	105	1250.00	987.02	143.85	2380.86	4876.01	9.36
	S_2	143	1000.00	1849.64	149.31	2998.95		S_2	119	1000.00	1419.10	76.05	2495.15		

increased value of stock levels from 107 to 119 when considering the lead-time of 1/12 years. The decrease in stock level indicates that we are trying to keep fewer inventories satisfying the shortages due to dependency with lower fill rates from 0.30 to 0.6. But with fill rate 0.65 and onwards, the stock level of dependent spare starts increasing from 89 to 98. It is because once the fill rate of spare part-1 increases, it also triggers the increase of inventory holding for second spare part to reduce the shortages. Suppose, with a review period 0.4 years, fill rate 0.3, and stock level of 107, we incur holding cost of Rs. 1148.85 and shortage cost Rs. 955.03. When dependency factor 0.3 is considered, inventory holding cost increases to Rs. 1248.97 and shortage cost reduces to Rs. 257.58 maintaining the same value of stock level. This shows that extra inventory is carried to reduce the shortage cost. We have also shown all the stock level values with respect to different values of α (0.1 to 0.99) and f (0.3 to 0.99) in Tables 8 and 9 with $L = 0$ and $L = 1/12$ years respectively for a common review period of 0.4 years. We have observed that order up-to level for first spare part remains constant and for the dependent spare part, it decreases with the increase of α value from 0.1 to 0.99. In the same way, total cost of both the spare parts decreases with the increase of the dependency factor α.

7. Case study

The proposed model is also applied to non-repairable spare parts of a case company where consumption records are regular and issued to specific maintenance activities. These data collected from the case company is a reputed public sector open cast mining company situated at the southern part of India. In this study, maintenance activities of belt conveyor and their major spare parts and dependent spare parts were identified using the procedure mentioned in Section 3.1. The details about these maintenance activities and spare parts are described in Table 10. Table 11 describes unit price, demand and dependency factor for the major and dependent spare parts. We have determined the order-up to level of major and minor spare parts using the computed dependency factor with the given fill rate and review period. We have assumed inventory holding cost and shortage cost as 20% and 50% of unit price respectively. The optimal order up-to level values are

Table 8
Order up-to level values with respect to fill rate and dependency factor ($L = 0$).

Review period $T = 0.4$ years											
f		$\alpha = 0.1$	$\alpha = 0.2$	$\alpha = 0.3$	$\alpha = 0.4$	$\alpha = 0.5$	$\alpha = 0.6$	$\alpha = 0.7$	$\alpha = 0.8$	$\alpha = 0.9$	$\alpha = 0.99$
0.30	S_1	24	24	24	24	24	24	24	24	24	24
	S_2	103	95	88	80	73	65	58	50	44	38
0.35	S_1	28	28	28	28	28	28	28	28	28	28
	S_2	103	95	88	80	72	65	57	51	45	40
0.40	S_1	32	32	32	32	32	32	32	32	32	32
	S_2	103	95	88	80	72	65	58	52	47	41
0.45	S_1	36	36	36	36	36	36	36	36	36	36
	S_2	103	95	88	80	73	66	59	54	48	42
0.50	S_1	40	40	40	40	40	40	40	40	40	40
	S_2	103	95	88	80	73	67	61	56	49	42
0.55	S_1	44	44	44	44	44	44	44	44	44	44
	S_2	103	95	88	81	74	69	63	57	49	42
0.60	S_1	48	48	48	48	48	48	48	48	48	48
	S_2	103	95	88	82	76	70	64	57	50	42
0.65	S_1	52	52	52	52	52	52	52	52	52	52
	S_2	103	96	89	83	78	72	65	58	50	42
0.70	S_1	56	56	56	56	56	56	56	56	56	56
	S_2	103	97	91	85	79	73	66	58	50	42
0.75	S_1	60	60	60	60	60	60	60	60	60	60
	S_2	104	98	92	87	81	74	66	58	50	42
0.80	S_1	64	64	64	64	64	64	64	64	64	64
	S_2	105	100	94	88	81	74	66	58	50	42
0.85	S_1	68	68	68	68	68	68	68	68	68	68
	S_2	107	101	96	89	82	74	66	58	50	42
0.90	S_1	73	73	73	73	73	73	73	73	73	73
	S_2	109	103	97	90	82	74	66	58	50	42
0.95	S_1	79	79	79	79	79	79	79	79	79	79
	S_2	110	104	98	90	82	74	66	58	50	42
0.99	S_1	88	88	88	88	88	88	88	88	88	88
	S_2	112	105	98	90	82	74	66	58	50	42

determined using our proposed algorithm for each spare part for three different fill rate of major spare parts such as 0.5, 0.7 and 0.9. Considering a review period of three months and lead time of one month, the summary of stock levels are given in Table 12.

We have observed that with higher fill rate of major spare, the order up-to levels of both major and minor spare parts are higher as compared to low fill rates. These stocks are maintained to reduce the shortage costs. When the dependency factor is more, the increase rate of stock level of dependent spare parts is more and vice versa. From Table 12, one can observe that for combination S61 and S62 and dependency factor 0.21, the rate of an increase of stock level of S62 is lower. On the other hand, S11and S12 with dependency factor 0.82, the increase rate of stock level of S12 is higher. The detail cost components of S91 and S92 are given in Table 13. One can see in Table 13 that total cost of major part decreases with the increase of fill rate. But the behavior of dependent spare S92 is different. The costs have their minimum at a fill rate 0.5. Higher costs are at 0.3, 0.7 and 0.9 fill rates. Lastly, we have made a comparison among stock levels and costs considering dependency and no dependency. Table 14 shows that considering no dependency or treating the spare S92 as an individual item, the optimum stock level is 295 units, and the total cost is Rs. 3322.52/year. But if we consider the dependency of 0.61 and fill rate of 0.5, the stock level is 155 units, and the total cost is Rs. 1921.02 per year. Hence, we have observed that we need to stock dependent spare parts at lesser quantity if the dependency among the spare parts is known.

7.1. Managerial insights

From this study, one can draw certain managerial insights for spare parts inventory management. None of the spare parts management studies have considered item-item relationship or dependency for inventory modeling. Our study introduces item dependency to inventory control modeling to determine optimal stock levels. The proposed methodology can help particularly a maintenance manager to know the exact relationship among the spares and suggest the optimal stock level of spares to maintain in inventory for future maintenance activities. The results of our numerical example indicate that the dependency factor has the

Table 9
Order up-to level values with respect to fill rate and dependency factor (L = 1/12 years).

Review period T = 0.4 years											
f		$\alpha = 0.1$	$\alpha = 0.2$	$\alpha = 0.3$	$\alpha = 0.4$	$\alpha = 0.5$	$\alpha = 0.6$	$\alpha = 0.7$	$\alpha = 0.8$	$\alpha = 0.9$	$\alpha = 0.99$
0.30	S_1	29	29	29	29	29	29	29	29	29	29
	S_2	126	116	107	98	89	79	70	61	53	46
0.35	S_1	33	33	33	33	33	33	33	33	33	33
	S_2	126	116	107	98	88	79	70	62	54	48
0.40	S_1	38	38	38	38	38	38	38	38	38	38
	S_2	126	116	107	98	88	79	71	63	57	50
0.45	S_1	43	43	43	43	43	43	43	43	43	43
	S_2	126	116	107	98	89	80	72	65	58	51
0.50	S_1	48	48	48	48	48	48	48	48	48	48
	S_2	126	116	107	98	89	81	74	67	59	51
0.55	S_1	53	53	53	53	53	53	53	53	53	53
	S_2	126	116	107	98	90	83	76	69	60	51
0.60	S_1	58	58	58	58	58	58	58	58	58	58
	S_2	126	116	107	99	92	85	78	69	60	51
0.65	S_1	62	62	62	62	62	62	62	62	62	62
	S_2	126	117	108	101	94	87	79	70	60	51
0.70	S_1	67	67	67	67	67	67	67	67	67	67
	S_2	126	117	110	103	96	88	79	70	60	51
0.75	S_1	72	72	72	72	72	72	72	72	72	72
	S_2	126	119	112	105	98	89	79	70	60	51
0.80	S_1	77	77	77	77	77	77	77	77	77	77
	S_2	128	121	114	107	99	89	80	70	60	51
0.85	S_1	82	82	82	82	82	82	82	82	82	82
	S_2	130	123	116	108	99	89	80	70	60	51
0.90	S_1	88	88	88	88	88	88	88	88	88	88
	S_2	132	125	117	109	99	89	80	70	60	51
0.95	S_1	95	95	95	95	95	95	95	95	95	95
	S_2	134	127	118	109	99	89	80	70	60	51
0.99	S_1	105	105	105	105	105	105	105	105	105	105
	S_2	135	128	119	109	99	89	80	70	60	51

Table 10
Different maintenance activities and spare parts.

SL.	Maintenance	Major spare part	Dependent spare part
1	Idler maintenance	Troughing idler (S11)	Idler Shaft (S12)
2	Belt maintenance	Belt Clamp (S21)	Belt Fasteners (S22)
3	Seal replacement	Dust seal (S31)	Grease seal (S32)
4	Power section	Protection relay (S41)	Solenoid valve (S42)
5	Noise reduction	Grease seal for return idler (S51)	Grease seal for troughing idler (S52)
6	Bearing replacement	Bearing housing (S61)	Labyrinth ring (S62)
7	Conveyor frame	Shiftable frame (S71)	Steel Track link (S72)
8	Hydraulic system	Hydraulic hose (S81)	Hydraulic jack oil seal (S82)
9	Loose fixing	Link bolt for troughing idler (S91)	Link bolt for return idler (S92)

direct effect on inventory holding and shortage cost of dependent spare parts. We have observed that when dependency factor is high (major and minor spare parts are strongly dependent), dependent (minor) spare parts will bear huge inventory carrying cost and less shortage cost if demand of major spare parts is satisfied with lesser fill rate. These costs also increase if we introduce the non-zero lead-time. Our model will also help a maintenance manager to determine exact stock levels of both major and minor spares by setting a service level like fill rate of major spare. The analysis also shows that the total inventory costs of two spare parts increases with the decrease of service level i.e. fill rate. By maintaining lower fill rate, major spare faces less inventory cost and very high shortage cost which in turn increases the inventory holding of the minor or dependent spares due to the dependency effect.

Table 11
Spare parts with unit price, demand and dependency factor.

SL.	Major spare	Price (Rs.)	Demand (Units/year)	Dependent spare	Price (Rs.)	Demand (Units/year)	Dependency factor
1	S11	600.00	150	S12	200.00	400	0.82
2	S21	5217.00	356	S22	35.60	800	0.56
3	S31	19.00	500	S32	12.50	1100	0.71
4	S41	345.00	589	S42	235.00	910	0.34
5	S51	12.50	534	S52	8.50	843	0.69
6	S61	945.56	80	S62	103.78	124	0.21
7	S71	8061.70	230	S72	448.20	847	0.91
8	S81	1287.50	348	S82	1173.25	430	0.42
9	S91	50.25	530	S92	24.50	670	0.61

Table 12
Order up-to level of major and dependent spare parts with different fill rate.

Dependency factor	Fill rate of major spare part →	0.5	0.7	0.9
	Spare parts	Order up-to levels (units)		
0.82	S11	31	44	58
	S12	124	125	129
0.56	S21	75	105	136
	S22	265	265	272
0.71	S31	106	148	191
	S32	355	356	365
0.34	S41	124	174	225
	S42	311	311	319
0.69	S51	113	158	204
	S52	210	217	217
0.21	S61	16	23	31
	S62	49	49	50
0.91	S71	48	68	88
	S72	280	280	288
0.42	S81	73	103	133
	S82	127	129	132
0.61	S91	112	157	202
	S92	155	161	161

Table 13
Stock, cost details of spare parts S91 and S92 with dependency factor 0.61.

Fill rate	Order up-to levels (units)	Ordering (Rs./year)	Holding (Rs./year)	Shortage (Rs./year)	Total cost (Rs./year)	Grand total cost (Rs./year)
0.30	67	1470.59	901.00	11597.06	13968.65	16050.33
	155	1176.47	833.09	72.12	2081.68	
0.50	112	1470.59	901.00	8288.24	10659.82	12580.85
	155	1176.47	672.43	72.12	1921.02	
0.7	157	1470.59	901.00	4979.41	7351.00	9325.27
	161	1176.47	767.84	29.96	1974.27	
0.9	202	1470.59	904.91	1699.34	4074.84	6199.08
	161	1176.47	917.81	29.96	2124.24	

Table 14
Stock, cost details of dependent spare S92 considering no dependency.

Order up-to levels (units)	Ordering (Rs./year)	Holding (Rs./year)	Shortage (Rs./year)	Total cost (Rs./year)
295	1176.47	2044.31	101.74	3322.52

8. Conclusion and scope for further research

In real-life applications, dependency among the demand of the items exists depending upon the valid relationship between them. For example, during maintenance operation, if one spare from an equipment and other from a different equipment is used at the same time, then one cannot say that these spares are dependent because there is no relation between these two spares in reality. Hence it is important to classify the spare groups and identify the frequent spares which are used together at the same time interval or at a predefined time sequence for similar type of maintenance activities. Most of the spare parts management studies have not considered the dependency factor in multi-item inventory control. Based on numerical examples, this study proves that stock level of spare parts can be reduced considerably considering dependency. Shortage of major spare parts can affect the holding and shortage cost of linked minor spare parts. Hence, stock of major spare parts should be maintained at higher fill rate in order to minimize the total inventory costs of both major and minor spares.

In this paper, we have studied dependent demands for two items using probabilistic demand. In future, this model can be extended for multiple items inventory control like one major spare with multiple dependent spares. If there is shortage of major spare, then all the associated spare parts demand will be affected i.e. shortage causes the blockage or more inventory carrying charges to the associated spares. Secondly, we have assumed the linear decrease rate of demand for minor item when major spare faces any shortage. But this may not be true always in real scenario. Hence one can extend the work by modeling the non-linear decrease rate of demand for minor spares. Further, we have proposed a periodic review policy which is applicable for only medium value and low value spares. As periodic review policy is not suitable for high-value spares, there is a scope to develop a continuous review model like $(S-1, S)$ policy considering dependent demand.

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